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Gavito, Valentin Francisco Jr.

THE FINITE ELEMENT METHOD APPLIED TO FLOWS IN TURBOMACHINES

Valentin Francisco Gavito

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

THE FINITE ELEMENT METHOD APPLIED TO FLOWS IN TURBOMACHINES

by

Valentin Francisco Gavito, Jr.

December 1976

Thesis Advisor: D.J. Collins

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of the NASA Task-1 stage transonic compressor and with numerical finite element results of Hirsch and Warzee.

THE FINITE ELEMENT METHOD APPLIED TO FLOWS IN TURBOMACHINES

by

Valentin Francisco Gavito, Jr.
Lieutenant, United States Navy
B.S.M.E., Southern Methodist University, 1970

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ABSTRACT

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I. INTRODUCTION

A. PROBLEM STATEMENT AND OBJECTIVE

The prediction of meridional flows within turbomachines, be they compressors or turbines, is a difficult but important part of the design process. The difficulty arises from the presence of three- dimensional and viscous effects within all turbomachines and the importance arises from the necessity to design accurately and efficiently.

To simplify the problem of viscous, three-dimensional analysis, Wu [Ref.1] showed that this complicated flow may be analyzed by solving two interrelated flows: one being the blade-to-blade flow describing the flow between rotating blades and the other being the meridional through flow which describes the radial equilibrium. These flows are depicted in Fig 1. In addition, an inviscid and axi symmetric assumption is made in the through-flow thereby simplifying the flow to a two-dimensional, axi symmetric, inviscid, and compressible analysis.

Three methods may be found in current reports regarding the solution of the radial equilibrium equation. The first two are the streamline curvature method [Ref.2,3,and 4] and the matrix method [Ref.5 and 6] which is basically a finite difference technique. The third, a relatively new method, is the finite element method. As shown by Hirsch and Warzee [Ref.7]´, the solution of the radial equilibrium equation by the finite element method is achieved by arranging the

equation for the stream function in quasi-harmonic form.

Due to the excellent results reported in Ref.7 and to further the research effort for finite element techniques in fluid flow problems, the purpose of this thesis is two fold. Firstly, the goal was to formulate a computer program for solution of the radial equilibrium equation paralleling the steps as presented by Hirsch and Warzee. Secondly, after suitable verification of computer results with those of Hirsch and Warzee, the goal was to compare computer predicted flows with measured performance data of the Naval Post Graduate School's transonic compressor.

The purpose of this paper is to present a report on the results obtained thus far. In Section II, the derivation of the radial equilibrium equation is presented followed by the application of the finite element method to this equation. Section III describes the computer program in some detail. Section IV contains selected test cases which were used in program testing and checking. In Section V, conclusions are presented along with recommendations for further study and work on the project. The appendices contain the program listing along with a sample test case for reference by the user. In addition, a list of references is contained for further reading on the subject of this paper.

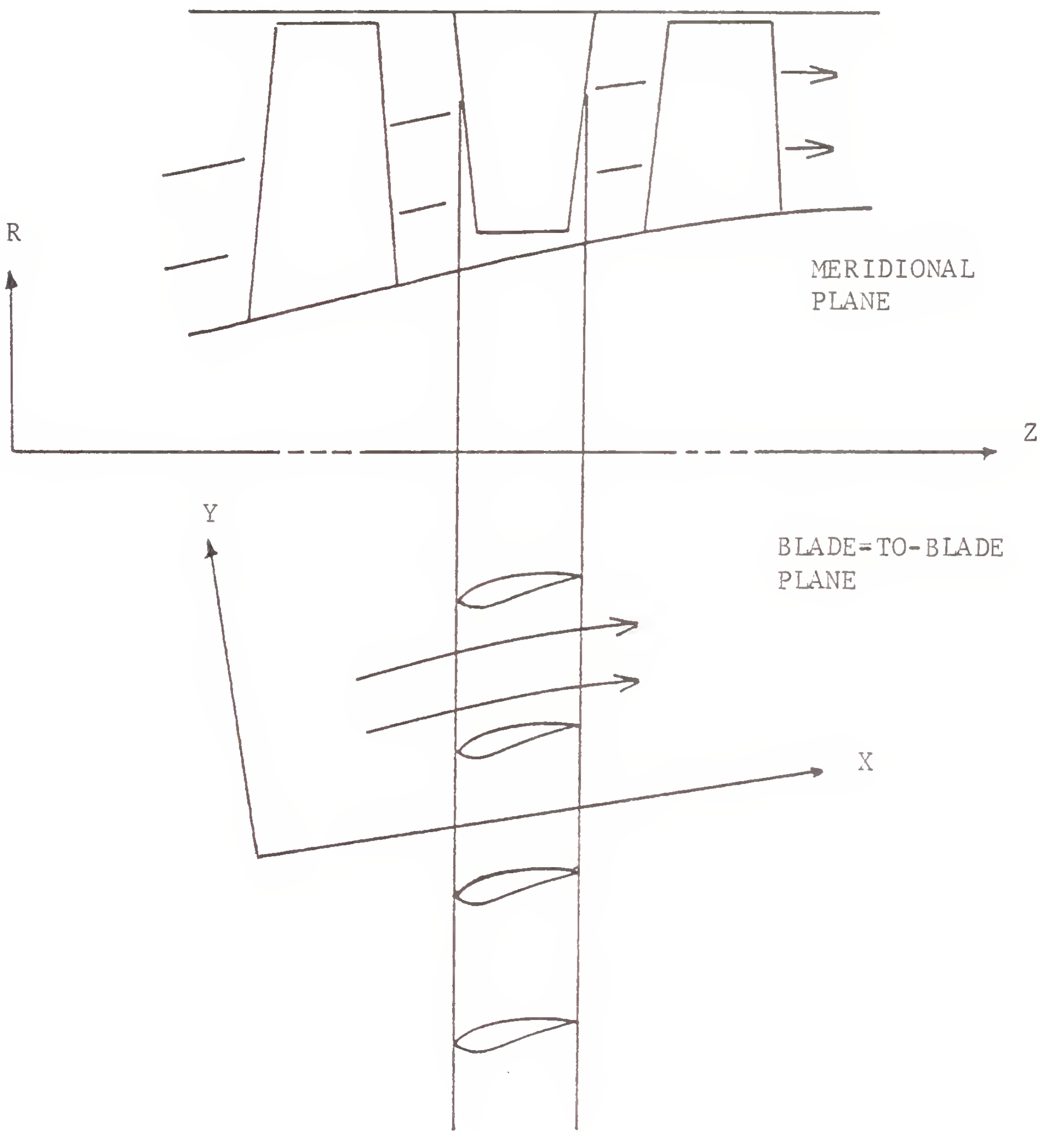


Figure 1 - MERIDIONAL AND BLADE-TO-BLADE PLANES

II. THEORY

A. THE DERIVATION OF THE RADIAL EQUILIBRIUM EQUATION

The following discussion is taken from Ref. 7 with slight changes in notation. The basic turbomachine geometry to be analyzed is depicted in Fig 2. Although the machine noted is one stage of a compressor, a similar analysis to the one that follows may be applied to other machines such as axial turbines and mixed-flow machines.

One begins with the Euler equation assuming the viscous forces to be negligible.

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = \nabla P / \rho \qquad \text{(II.A.1)}$$

The continuity equation, assuming unsteady flow is,

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \vec{V}\right) = 0 \qquad \text{(II.A.2)}$$

The First Law of thermodynamics in a fluid field becomes,

$$T \nabla s = \nabla h - \nabla P / \rho \qquad \text{(II.A.3)}$$

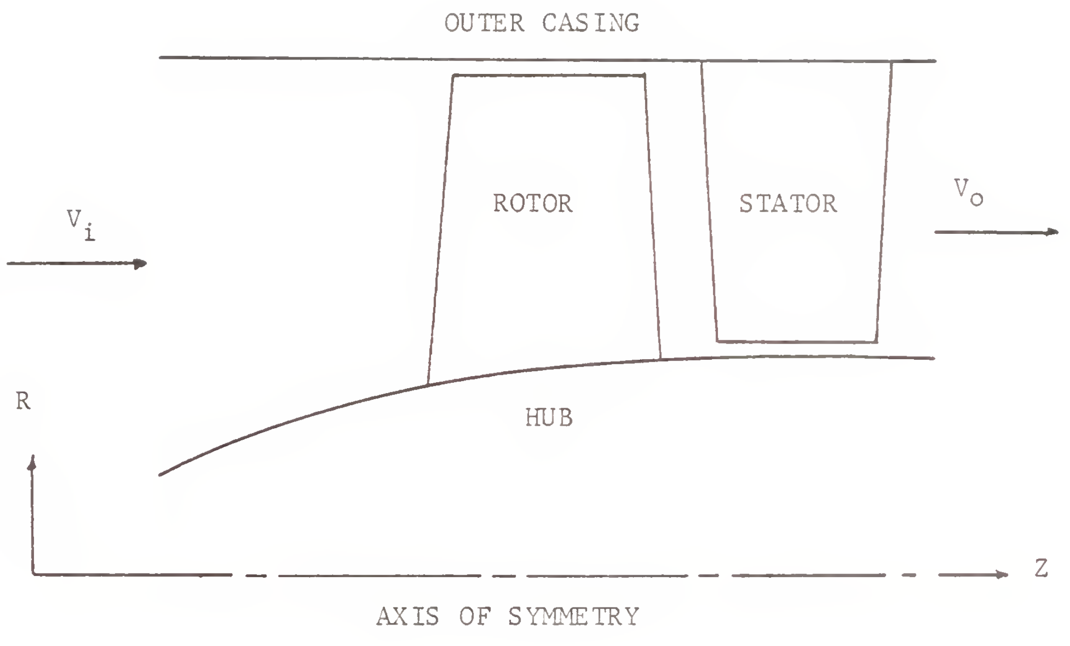


Figure 2 - TURBOMACHINE GEOMETRY

Substituting equation (II.A.3) into equation (II.A.1) leads to the Crocco equation,

$$\frac{\partial \vec{V}}{\partial t} - \vec{V} \times (\nabla \times \vec{V}) = T\nabla S - \nabla H \qquad \text{(II.A.4)}$$

where H is the total enthalpy.

Assuming a steady and adiabatic flow, the energy equation becomes simply,

$$(\vec{V} \cdot \nabla) H = 0 \qquad \text{(II.A.5)}$$

which shows that along a streamline in a stationary system, the total enthalpy is constant.

In a relative system, such as the case in a rotor blade row, the total relative velocity, $\vec{W}$, can be expressed in the following form,

$$\vec{W} = \vec{V} + \vec{\omega} \times \vec{R} = \vec{V} + \vec{u} \qquad \text{(II.A.6)}$$

where $\vec{\omega}$ is the constant angular velocity and $\vec{U}$ is the constant peripheral speed of the relative system.

Now, the Crocco equation in a relative system becomes,

$$\frac{\partial \vec{W}}{\partial t} - \vec{W} \times (\nabla \times \vec{W}) = T\nabla S - \nabla\left(h + \frac{W^2}{2} - \frac{\omega^2 R^2}{2}\right) \qquad \text{(II.A.7)}$$

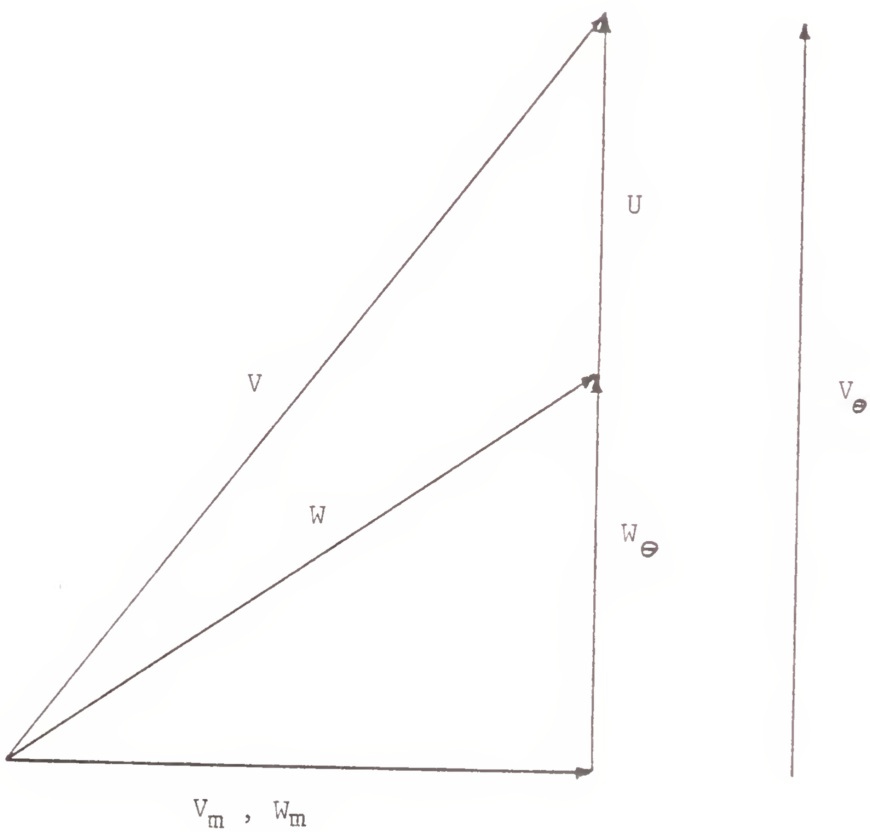
Parallel to equation (II.A.5) for the stationary system, the energy equation, assuming steady and adiabatic (relative) flow in a relative system, becomes

$$(\vec{W} \cdot \nabla) H_R = 0 \tag{II.A.8}$$

where H_R is the relative total enthalpy expresed as follows,

$$H_R = h + \frac{W^2}{2} - \frac{\omega^2 R^2}{2} \tag{II.A.9}$$

From the following velocity diagram,



equation (II.A.9) may be arranged as follows.

Since,

$$W_m^2 + W_\theta^2 = W^2 = V_m^2 + (V_\theta - U)^2 \tag{II.A.10}$$

then,

$$W^2 = V_m^2 + V_\theta^2 - 2UV_\theta + U^2 \tag{II.A.11}$$

and,

$$W^2 = V^2 + U^2 - 2UV_\theta \tag{II.A.12}$$

Substituting equation (II.A.12) into equation (II.A.9) leads to the following relation,

$$H_R = h + \frac{V^2}{2} - UV_\theta = H - UV_\theta \tag{II.A.13}$$

Equation (II.A.8) shows that H_R is constant along a streamline in a relative system.

Upon integrating equation (II.A.8) between the rotor inlet and outlet, the Euler equation for turbomachines is found,

$$\Delta H \Big|_{IN}^{OUT} = \Delta \left(\vec{U} \cdot V_\theta \right) \Big|_{IN}^{OUT} \tag{II.A.14}$$

It may be shown [Ref.9] that by circumferentially averaging equation (II.A.1), and under the axi symmetric flow assumption the following relation is valid,

$$-\vec{V}\times(\nabla\times\vec{V}) = T\cdot\nabla S - \nabla H + F_b + F_d \qquad \text{(II.A.15)}$$

where F_b is the body force of the blades acting on the fluid and all variables are mean values along the direction of the circumference. Hence equation (II.A.15) is an approximation for axi symmetric flow. As a final note on equation (II.A.15), since the viscous forces were neglected in equation (II.A.1), there must be a force introducing the entropy variations along the blade. This force is proportional to the pressure loss coefficient and is labeled F_d, the dissipative force. F_d produces work which in turn produces entropy production radially along the blade. Under the axi symmetric assumption, entropy varies axially and radially only and is assumed to be proportional to the pressure loss coefficients [Ref. 2 and 8].

Due to boundary conditions imposed on the problem and the axi symmetric assumption, cylindrical coordinates, (r,θ,z), will be used in all subsequent analysis. Therefore, equation (II.A.15), in cylindrical coordinates and with axial symmetry is as follows,

$$\frac{V_\theta}{R}\frac{\partial}{\partial R}(RV_\theta) - V_z\left(\frac{\partial}{\partial z}V_R - \frac{\partial}{\partial R}V_z\right) = \frac{\partial H}{\partial R} - T\frac{\partial S}{\partial R} - F_{br} - F_{dr} \qquad \text{(II.A.16)}$$

$$\frac{V_z}{R}\frac{\partial}{\partial z}(RV_\theta) + \frac{V_R}{R}\frac{\partial}{\partial R}(RV_\theta) = F_\theta \qquad \text{(II.A.17)}$$

$$V_R\left(\frac{\partial}{\partial Z}V_R - \frac{\partial}{\partial R}V_Z\right) - \frac{V_\theta}{R}\frac{\partial}{\partial Z}(RV_\theta) = \frac{\partial H}{\partial Z} - T\frac{\partial S}{\partial Z} - F_Z \qquad \text{(II.A.18)}$$

It is important to note here that under the axi symmetric assumptions, equation (II.A.15) reduces to the following,

$$\vec{V}\cdot\vec{F}_b = 0 \qquad \text{(II.A.19)}$$

Likewise in a relative system (rotor), the axi symmetric assumption leads to the following,

$$\vec{W}\cdot\vec{F}_b = 0 \qquad \text{(II.A.20)}$$

Equation (II.A.16) describes the meridional through flow radial equilibrium equation for the finite element method. Since one is concerned with the meridional plane, the following derivative expression is taken from Fig 3.

$$V_m\frac{\partial}{\partial m} = V_R\frac{\partial}{\partial R} + V_Z\frac{\partial}{\partial Z} \qquad \text{(II.A.21)}$$

Therefore equation (II.A.17) reduces to,

$$RF_\theta = V_m\frac{\partial}{\partial m}(RV_\theta) \qquad \text{(II.A.22)}$$

which reveals that in a duct where there are no blades and therefore no blade forces, angular momentum is constant

along a streamline. In that case,

$$\frac{\partial}{\partial m}\left(R V_\theta\right) = 0 \qquad \text{(II.A.23)}$$

As shown in Ref.9, the circumferentially averaged continuity equation is the following,

$$\frac{\partial}{\partial R}\left(\rho R b V_R\right) + \frac{\partial}{\partial z}\left(\rho R b V_z\right) = 0 \qquad \text{(II.A.24)}$$

where b is the blockage factor defined by Hirsch and Warzee as the tangential area reduction due to the thickness of the blade.

$$b = 1 - \frac{t}{s} \qquad \text{(II.A.25)}$$

where t is blade thickness and s is blade spacing.

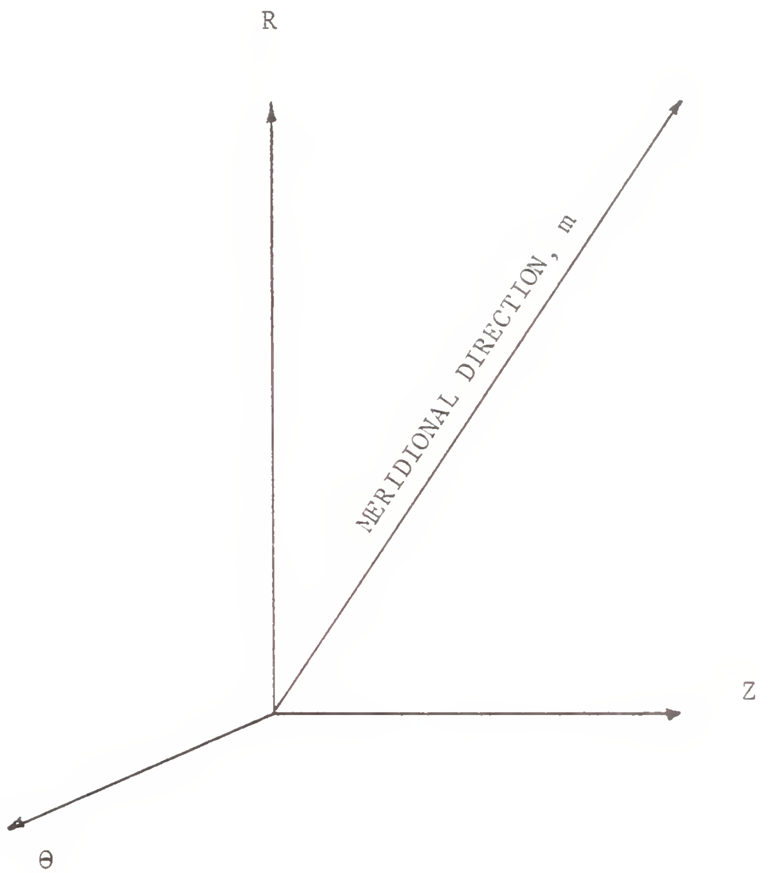


Figure 3 - MERIDIONAL PLANE

One further step in the formulation of the radial equilibrium equation for solution by the finite element method involves introducing the stream function. In cylindrical coordinates, the stream functions are defined as follows,

$$V_z = \frac{1}{\rho R b}\frac{\partial \psi}{\partial R} \qquad \text{(II.A.26)}$$

$$V_R = -\frac{1}{\rho R b}\frac{\partial \psi}{\partial z} \qquad \text{(II.A.27)}$$

Substituting these expressions into equation (II.A.16), the equation becomes,

$$\frac{\partial}{\partial R}\left(\frac{1}{\rho R b}\frac{\partial \psi}{\partial R}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho R b}\frac{\partial \psi}{\partial z}\right) = \frac{1}{V_z}\left[\frac{\partial H}{\partial R} - T\frac{\partial s}{\partial R} - \frac{V_\theta}{R}\frac{\partial}{\partial R}(R V_\theta) - F_{br} - F_{dr}\right] \qquad \text{(II.A.28)}$$

The right hand side of equation (II.A.28) is applicable to the absolute flows in the stator and duct regions. For relative flows such as those in the rotor, the right hand side is modified by replacing the total enthalpy, H, by the relative total enthalpy, H_R, and the quantity V_θ/R is replaced by W_θ/R.

As a last assumption in the formulation of the governing relation for the meridional through flow radial equilibrium equation, both the radial component of the body force, F_b and the radial component of the dissipative force, F_d, are neglected. This assumption, [Ref.1,8] does not hamper the accuracy of the results for conditions at design speed. Even though published compressor performance data used for

the test case in this thesis was obtained at 0.5 design speed, these force terms were also neglected in the computer program. As will be shown later, this assumption could possibly have had adverse effects on the predicted axial velocity profiles at the rotor hub and tip regions.

The final representation of the meridional radial equilibrium equation to be solved by the finite element method is as follows,

$$\frac{\partial}{\partial R}\left(k\frac{\partial\psi}{\partial R}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial\psi}{\partial z}\right)+f=0 \qquad \text{(II.A.29)}$$

where,

$$k=\frac{1}{\rho R b} \qquad \text{(II.A.30)}$$

and

$$f=\frac{1}{V_z}\left[T\frac{\partial s}{\partial R}-\frac{\partial H}{\partial R}+\frac{V_\theta}{R}\frac{\partial}{\partial R}\left(RV_\theta\right)\right] \qquad \text{(II.A.31)}$$

B. THE FINITE ELEMENT METHOD APPLIED TO THE RADIAL EQUILIBRIUM EQUATION

In order to formulate equations (II.A.29) through (II.A.31) in matrix form for solution by the finite element method, one must apply a weighted residual technique to the equations for numerical solution. The weighted residual method used here is the Galerkin's Method. The following discussion is taken from Ref. 7 with only slight changes in notation.

Rewriting equation (II.A.29) and dividing through by R, one has,

$$\frac{1}{R}\left\{\frac{\partial}{\partial R}\left(K\frac{\partial\psi}{\partial R}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right)+f\right\}=0 \tag{II.B.1}$$

where this equation represents the flow in the volume, V.

The boundary condition for this partial differential equation, after dividing through by R, is,

$$\frac{1}{R}\left\{K\frac{\partial\psi}{\partial n}+\alpha_1\left(\psi-\psi_0\right)\right\}=0 \tag{II.B.2}$$

where this equation solves the flow on the closed boundary of the volume, or, S.

By applying the weighted residual process to equations (II.B.1) and (II.B.2) and using an arbitrary weighting function, W (r,z), one has

$$\int_V W(R,z)\, r_{VOL}\, dV+\int_S W(R,z)\, r_{SUR}\, dS'=0 \tag{II.B.3}$$

where r_{VOL} and r_{SUR} are the volume and surface residuals respectively, or,

$$r_{VOL}=-\frac{1}{R}\left\{\frac{\partial}{\partial R}\left(K\frac{\partial\psi}{\partial R}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right)+f\right\}=0 \tag{II.B.4}$$

$$r_{SUR}=\frac{1}{R}\left\{K\frac{\partial\psi}{\partial n}+\alpha_1\left(\psi-\psi_0\right)\right\} \tag{II.B.5}$$

If the solution to equation (II.B.1) was exact, both r_{VOL} and r_{SUR} would be equal to zero.

In order to clarify the boundary condition, equation (II.B.2), one may analyze the equation as follows.

On the surface, S, where ψ is specified,

$$\psi = \psi_0 \qquad \text{(II.B.6)}$$

and,

$$\alpha_1 \to \infty \qquad \text{(II.B.7)}$$

Similarly, on the surface, where $\frac{\partial \psi}{\partial n} = 0$, S_2, where

$$\alpha_1 = 0 \qquad \text{(II.B.8)}$$

$$S_1 \cap S_2 = 0, \quad S_1 \cup S_2 = S \qquad \text{(II.B.9)}$$

Due to the axi symmetric assumption, the final equation will not involve dV and dS but the intersection of dV and dS with the meridional plane. Therefore, one must transform the volume integral, dV, to a surface integral and the surface integral, dS, to a line integral.

Hence, let,

$d\Omega$ = intersection of dV and meridional plane

dC = intersection of dS and meridional plane

and,

$$dV = 2\pi R \, d\Omega$$

$$dS = 2\pi R \, dC$$

With this transformation, one may rewrite equation

(II.B.3) as follows,

$$\int_{\Omega} -W(R,z)\left[\frac{\partial}{\partial R}\left(K\frac{\partial\psi}{\partial R}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right) + f\right] 2\pi\, d\Omega + \int_{C} W(R,z) K \frac{\partial\psi}{\partial n} 2\pi\, dC = 0 \qquad \text{(II.B.10)}$$

where on the contour, $C_1, \psi = \psi_0$.

One must now integrate the first term in equation (II.B.10) by parts to obtain the following,

$$-\int_{\Omega} W\left[\frac{\partial}{\partial R}\left(K\frac{\partial\psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right)\right] 2\pi\, d\Omega - \int_{\Omega} W\cdot f\, d\Omega + \int_{\Omega} K\left[\frac{\partial\psi}{\partial R}\frac{\partial W}{\partial R} + \frac{\partial\psi}{\partial z}\frac{\partial W}{\partial z}\right] d\Omega + \int_{C} WK\frac{\partial\psi}{\partial n}\cdot 2\pi\, dC = 0 \qquad \text{(II.B.11)}$$

Inspecting the first term in equation (II.B.11), one may use the following integral theorem to simplify further

$$\int_{\Omega} \partial_{\beta}\phi\, d\Omega = \int_{C} \phi\, n_{\beta}\, dC \qquad \text{(II.B.12)}$$

Rewriting equation (II.B.11) ,gives,

$$-\int_{C} WK\left[\frac{\partial\psi}{\partial R} n_R + \frac{\partial\psi}{\partial z} n_z\right] dC - \int_{C} Wf\, d\Omega + \int_{\Omega} K\left[\frac{\partial\psi}{\partial R}\frac{\partial W}{\partial R} + \frac{\partial\psi}{\partial z}\frac{\partial W}{\partial z}\right] d\Omega + \int_{C} WK\frac{\partial\psi}{\partial n} 2\pi\, dC = 0 \qquad \text{(II.B.13)}$$

Finally, since

$$\frac{\partial\psi}{\partial n} = \frac{\partial\psi}{\partial R} n_R + \frac{\partial\psi}{\partial z} n_z \qquad \text{(II.B.14)}$$

equation (II.B.13) reduces to the following,

$$\int_{\Omega}\left[K\left(\frac{\partial\Psi}{\partial R}\frac{\partial W}{\partial R}+\frac{\partial\Psi}{\partial z}\frac{\partial W}{\partial z}\right)-fW\right]d\Omega=0 \qquad \text{(II.B.15)}$$

One now has the final equation in the form for use by the weighted residual method using any arbitrary weighting function, $W(r,z)$. As noted previously, the Galerkin's Method will be used here which implies that the weighting functions are the same functions used in approximating the stream function, Ψ.

Before applying the finite element method, one must discretize the continuum and then approximate the unknown function, Ψ, by a set of polynomials. For this particular problem, eight-noded iso-parametric elements were chosen for discretization, see Fig 4, and the following approximating functions were used.

$$\Psi=\sum_{i=1}^{8}N_i(\xi,\eta)\Psi_i \qquad \text{(II.B.16)}$$

where,

$N_i(\xi,\eta)$ = shape functions

Ψ_i = value of Ψ at the node

Ψ = value of Ψ at any arbitrary location within the element. The shape functions, N_i, used here are defined by the following relations as shown in Ref.10,

$$N_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)$$

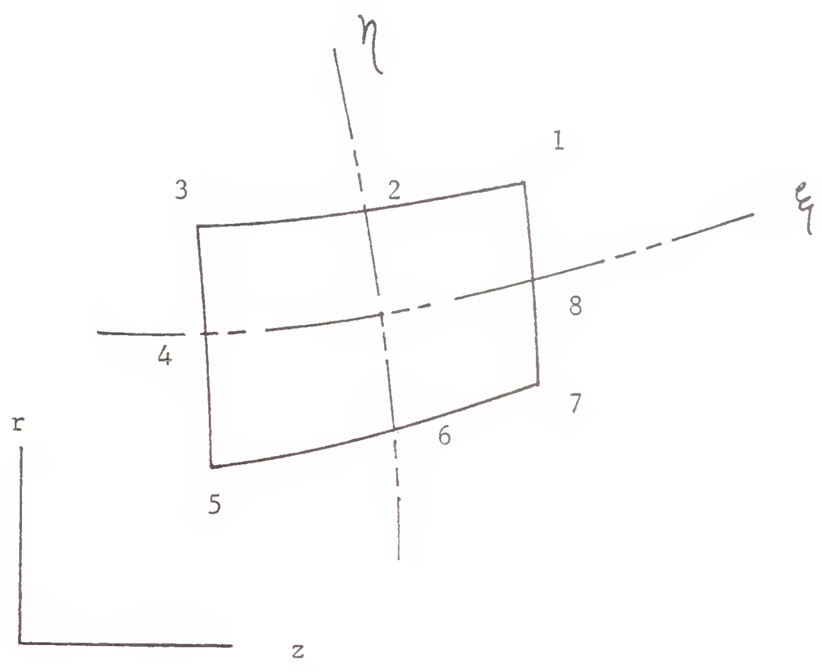
$$N_i(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1+\eta\eta_i) \qquad \text{(II.B.17)}$$

$$N_i(\xi,\eta) = \frac{1}{2}(1+\xi\xi_i)(1-\eta^2)$$

where the following coordinate transformations are used,

$$r = \sum_{i=1}^{8} N_i(\xi,\eta)\, r_i \qquad \text{(II.B.18)}$$

$$z = \sum_{i=1}^{8} N_i(\xi,\eta)\, z_i$$



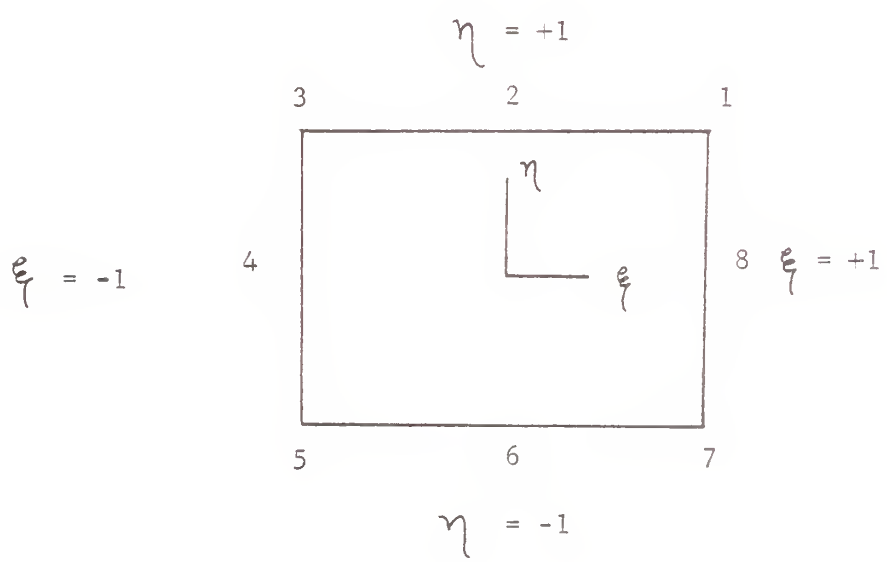


Figure 4 - ISOPARAMETRIC QUADRILATERAL ELEMENT

At this point, one is ready to apply the Galerkin Method to equation (II.B.15) by substituting equation (II.B.16) for the unknown function Ψ, and N_i for the weight function, W, which yields,

$$\int_{\Omega} K \left\{ \frac{\partial N_i}{\partial r} \sum_{i=1}^{8} \Psi_i \left(\frac{\partial N_j}{\partial r} \right) + \frac{\partial N_i}{\partial z} \sum_{i=1}^{8} \Psi_i \left(\frac{\partial N_i}{\partial z} \right) \right\} d\Omega - \int_{\Omega} f_i N_i \, d\Omega = 0 \quad \text{(II.B.19)}$$

This integration yields the following system of equations which is solved for the unknown nodal Ψ,

$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ \vdots & & & \\ K_{n1} & \cdots & & K_{nn} \end{bmatrix} \begin{Bmatrix} \Psi_1 \\ \vdots \\ \Psi_n \end{Bmatrix} = \begin{Bmatrix} f_1 \\ \vdots \\ f_n \end{Bmatrix} \quad \text{(II.B.20)}$$

where,

$$K_{ij} = \int_{\Omega} k \left\{ \frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right\} d\Omega \quad \text{(II.B.21)}$$

and,

$$f_i = \int_{\Omega} f \cdot N_i \, d\Omega \quad \text{(II.B.22)}$$

In addition since both the 'stiffness matrix', K, and the right hand side vector, [F], are functions of Ψ, the system as defined by equations (II.B.20) through (II.B.21) must be solved iteratively.

At this point one has the total finite element formulation of the radial equilibrium equation as defined by equations (II.B.19) and (II.B.20). The problems which remain to be clarified are basically two fold. Firstly one

must evaluate the integrals in equations (II.B.19) and (II.B.2) by numerical methods, and secondly, the solution procedure for the non-linearity must be formulated. In Part C, both of these final steps are presented.

C. NUMERICAL INTEGRATION OF STIFFNESS MATRIX AND SOLUTION PROCEDURE

1. Numerical integration of the stiffness matrix

As noted in Section II.B, evaluation of equation (II.B.21) must be performed numerically. In addition, one realizes that the derivative expressions enclosed within the interval must be evaluated by a coordinate transformation. This is done in the following way,

Since,

$$r = \sum_{i=1}^{8} N_i(\xi,\eta)\, r_i$$

$$z = \sum_{i=1}^{8} N_i(\xi,\eta)\, z_i \qquad \text{(II.C.1)}$$

then,

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial z}\frac{\partial z}{\partial \xi} + \frac{\partial N_i}{\partial r}\frac{\partial r}{\partial \xi}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial z}\frac{\partial z}{\partial \eta} + \frac{\partial N_i}{\partial r}\frac{\partial r}{\partial \eta} \qquad \text{(II.C.2)}$$

and in matrix form,

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial z}{\partial \xi} & \frac{\partial r}{\partial \xi} \\ \frac{\partial z}{\partial \eta} & \frac{\partial r}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial r} \end{Bmatrix} \tag{II.C.3}$$

Furthermore, defining the Jacobian matrix as,

$$J = \begin{bmatrix} \frac{\partial z}{\partial \xi} & \frac{\partial r}{\partial \xi} \\ \frac{\partial z}{\partial \eta} & \frac{\partial r}{\partial \eta} \end{bmatrix} \tag{II.C.4}$$

then by dividing both sides of equation (II.C.3) by J, one has the following transformation,

$$\begin{Bmatrix} \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial r} \end{Bmatrix} = \left[J\right]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \tag{II.C.5}$$

In addition, it has been shown [Ref.9] that

$$dz\,dr = |J|\, d\xi\, d\eta \tag{II.C.6}$$

Now, with equations (II.C.5) and (II.C.6), equation (II.B.21) becomes the following,

$$K_{ij} = \int_{-1}^{1}\int_{-1}^{1} K \left\lfloor \frac{\partial N_i}{\partial \xi} \; \frac{\partial N_i}{\partial \eta} \right\rfloor \left\{ [J]^{-1} \right\}^T [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \det [J] \, d\xi \, d\eta \qquad \text{(II.C.7)}$$

Equation (II.C.7) is best integrated using the Gauss-Legendre integration method since it is of the following form,

$$K_{ij} = \int_{-1}^{1}\int_{-1}^{1} G(\xi, \eta) \, d\xi \, d\eta \qquad \text{(II.C.8)}$$

or finally, [Ref.10],

$$K_{ij} = \sum_{i=1}^{2} \left\{ A_i B_i \, f(\xi_i, \eta_i) \right\} \qquad \text{(II.C.9)}$$

where A_i and B_i are coefficients (Fig 5) for both two and three point Gaussian Quadrature.

At this point, one has the tools to calculate all the elements of the stiffness matrix. In like manner, the right hand side vector, f, is calcualted by numerical integration.

NUMBER OF GAUSSIAN POINTS	$\pm\eta$ $\pm\xi$	$\pm A_i$ $\pm B_i$
2	0.57735 02691	1.00000 00000
3	0.77459 66692 0.00000 00000	0.55555 55555 0.88888 88888

Figure 5 - GAUSSIAN INTEGRATION POINTS

2. Solution procedure

The following is a synopsis of the basic solution process. Specific details concerning equations and methods of computer coding are covered in the proceeding section. The proceeding is meant to give the reader a preview of the solution process.

a. Discretization

Initially the machine under analysis is discretized into eight-node iso-parametric elements. The axial calculation stations are placed arbitrarily in the duct regions and along blade edges and centers for the rotor and stator as shown in Fig 6. At this point the system topology and nodal coordinates are specified.

b. Initialization

To begin the iteration process, one must assume an initial internal stream function, velocity, and density distribution. In the program, the initial internal stream function was assumed to be that of the outer boundary throughout while the velocity and density distribution was assumed to be that of the inlet.

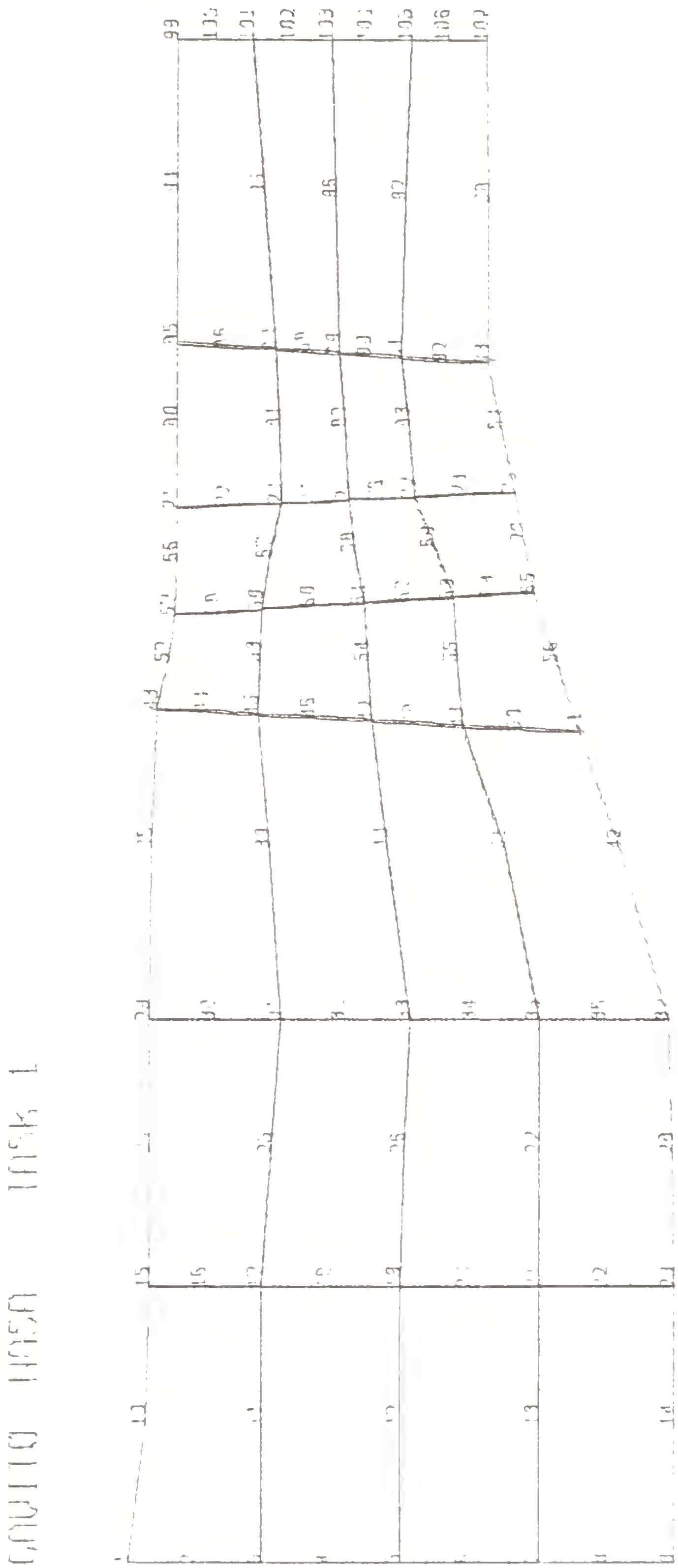


Figure 6 - COMPRESSOR DISCRETIZATION

c. Calculation of thermodynamic variables

Before calculating the right-hand side vector, f, one must obtain distributions of angular momentum, enthalpy, and entropy. This is done by first calculating the thermodynamic variables at the inlet axial station from the given inlet conditions. In order to proceed axially through the machine to calculate the nodal angular momentum, enthalpy, and entropy, the following three equations derived in Section III are used.

$$H = C_p T \quad = \text{constant along a stator streamline}$$

$$H_R = C_p T_{tr} - \frac{(\omega r)^2}{2} = \text{constant along a rotor streamline}$$

$$r V_\theta = \text{constant along a duct streamline}$$

An example of this calculation procedure for the duct region is shown graphically in Fig 7.

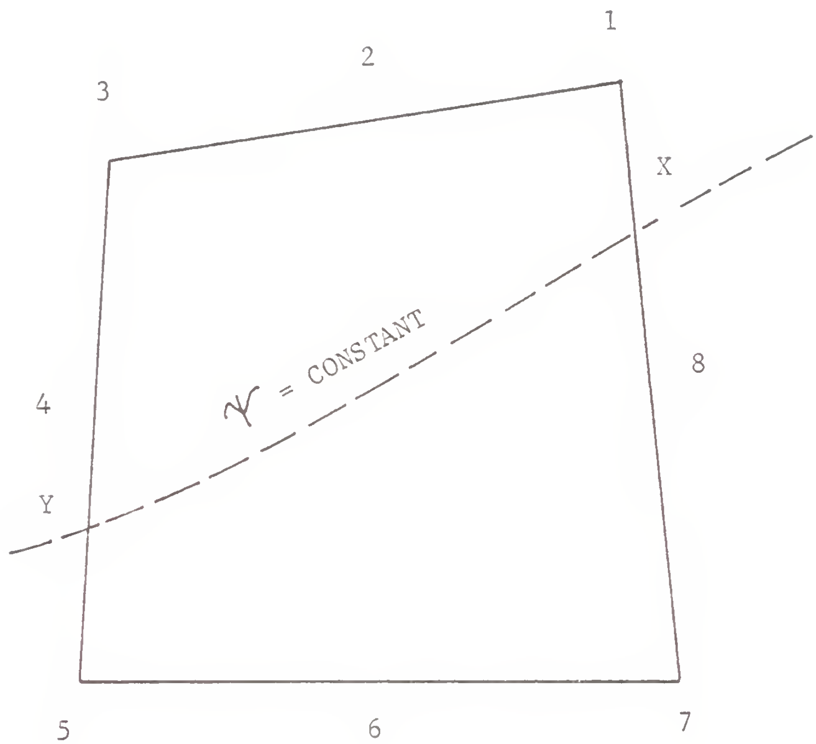


Figure 7 - DUCT ELEMENT

In this figure, the angular momentum at point X is equal to the angular momentum at point Y. More formally,

$$(rV_\theta)_X = \sum_{i=3}^{5} N_i(\xi,\eta)\,(rV_\theta)_i = (rV_\theta)_Y \qquad \text{(II.C.2.1)}$$

Since the previous axial station's thermodynamic variables are known, one must now find the values of ξ and η at point Y. This is done iteratively in the following way. Since

$$\psi\big|_Y = \psi\big|_X = \sum_{i=3}^{5} N_i(\xi,\eta)\,\psi_i \qquad \text{(II.C.2.2)}$$

and along the left side of the element,

$$\xi = -1 \qquad \text{(II.C.2.3)}$$

then equation (II.C.2.2) may be solved for η by a suitable iteration method. As will be shown in the next section, a half-interval method was used to obtain the unknown η . Once η is known, then equation (II.C.2.1) is solved for the angular momentum at point X. The rotor and stator are handled in a similar fashion. In addition, the rotor and stator deviate the flow creating a three-dimensional flow field between the blades in the respective blade row. Low speed cascade correlation data [Ref.13] was used to calculate the effective turning angles in the rotor and stator. These effects are calculated beforehand with known mass flow rate and uniform axial velocity assumptions at the rotor inlet. The results of these calculations are part of the input data routine in the form of relative and absolute flow angles at the rotor nodes and absolute flow angles at

the stator nodes. This will be shown more exactly in the next section.

d. Calculate matrices

At this point, the right hand side vector, f, and the stiffness matrix, K, are calculated.

e. Solve system of equations

The system of equations as shown in equation (II.B.20) is solved for the nodal stream function.

f. Perform relaxation iteration

Due to the strong non-linear properties of the system of of equations, the following iterative scheme is necessary.

$$\psi_i^{n+1} = \psi_i^n + \alpha \left[\hat{\psi}_i^{n+1} - \psi_i^n \right] \qquad \text{(II.C.2.4)}$$

where α is the under relaxation factor. As will be shown in Section III, this scheme is performed only in certain regions of the machine and in addition after a specified number of iterations.

g. Update velocity and density profiles

Using the current nodal distribution of the stream function, axial and radial nodal velocity components are calculated along with a new nodal density distribution.

Again, this calculation procedure will be shown in the next section.

h. Test for convergence of ψ

Stream function convergence criteria is now tested and will determine if further iterations are necessary. The solution is said to converge if the following equation holds for all nodes.

$$\left| \frac{\psi_i^{n} - \psi_i^{n+1}}{\psi_i^{n+1}} \right| < \epsilon \qquad \text{(II.C.2.5)}$$

where ϵ is a designated requirement for convergence.

i. Summary

In summary, the eight steps involved in the solution are noted below;

(1) Discretize the continuum.

(2) Assume an initial stream function, velocity, and density solution.

(3) Calculate the nodal thermodynamic variables from the given inlet conditions.

(4) Form the right hand side vector, f(r,z), and the stiffness matrix, K.

(5) Solve the system of equations, given by, [K] = [F] for a new stream function distribution.

(6) Perform relaxation iteration if required.

(7) Calculate new nodal velocity and density distributions from the current stream function solution.

(8) Test the solution for convergence, and if required, repeat steps (3) through (8) using the current nodal stream function values.

This concludes the solution description and now one is ready to more completely understand the computer program which assembles the preceding eight steps.

III. THE PROGRAM

A. OVERALL FLOWCHART AND DESCRIPTION

The overall flowchart of the program is depicted in Fig 8. Those blocks denoted by the letter 'S' are subroutines, while the remaining calculations are an integral part of the main program.

After proper dimensioning of all arrays and subsequent initialization, the input data are read and then printed. This not only presents a physical picture of the problem but also serves as a cross check to the user for correct data insertion. In addition, a subroutine is available to obtain a computer drawn plot of the mesh (Fig 6) and is a further check on proper data input.

At this point all the necessary variables have been stored and the iteration counter for stream function convergence is set. With the current nodal values of ψ and the given inlet thermodynamic conditions, the thermodynamic variables throughout the machine are calculated. From the calculated values of enthalpy and angular momentum, (isentropic flow is assumed), the right-hand side vector is calculated followed by the stiffness matrix calculation (equation II.B.21).

The system of equations (equation (II.B.20)) is now solved for the new nodal stream function distribution. It is here where for all iterations but the first that a

relaxation factor is applied as noted previously in equation (II.C.2.4). The reasoning behind not applying the relaxation scheme to the value of nodal ψ after the first iteration is the fact that the first iteration produced a close approximation to the correct stream function distribution. With this close approximation to the stream function came a velocity and density distribution which in turn was near the correct solution. It was found that if the first iteration was relaxed, the second iteration became unstable since in fact the velocities and densities were themselves farther from the true values than were assumed initially.

After testing the nodal stream function for convergence by use of equation (II.C.2.5), the calculation process is either repeated or ceased by virtue of convergence or limiting the number of iterations.

As stated previously, low speed cascade correlation data [Ref.13] were used to calculate turning angles in the blade regions. These angles were assumed constant throughout the solution and not refined after subsequent iterations. Further work on the computer program could entail an additional computational routine which would calculate the new turning angles after each iteration. A sample calculation of rotor turning angles is shown in Appendix D.

In the following sections the program structure is examined in more detail.

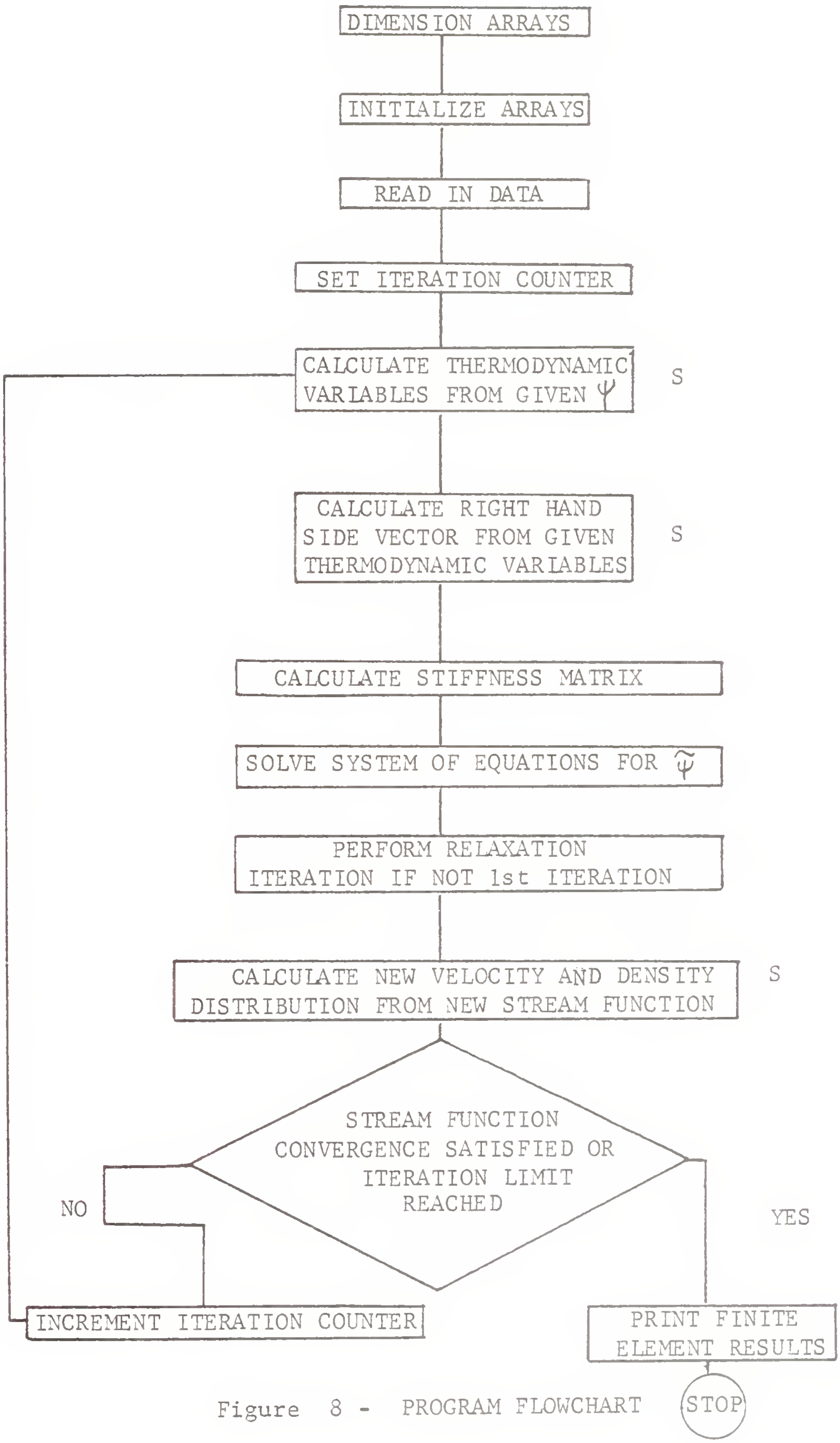


Figure 8 - PROGRAM FLOWCHART

B. THE MAIN PROGRAM

1. The input routine

The following is a description of the input data required by the program. The data are arranged into twelve categories described in the following manner.

a. category 1

Problem identification.

b. category 2

Number of nodes and number of elements.

c. category 3

Node numbers, nodal coordinates and nodal blockage factor.

d. category 4

System topology.

e. category 5

Element type; duct, rotor, or stator.

f. category 6

Absolute flow angles for rotor and stator nodes.

g. category 7

Relative flow angles for rotor nodes.

h. category 8

Inlet thermodynamic quantities.

i. category 9

Physical constants for fluid under observation.

j. category 10

First estimate of internal stream function.

k. category 11

Node numbers and specified nodal stream function.

l. category 12

Node numbers where the right hand side, f(r,z), is to be calculated.

Before describing in detail the format to be followed for data insertion, it is important to note the following assumptions.

(1) Uniform flow conditions at inlet and outlet.

(2) Uniform flow conditions at rotor inlet for calculation of appropriate turning angles. This assumption is necessary to calculate the values of rotor and stator flow angles.

With this in mind , the discussion will continue.

The following describes each category in more detail.

Category 1:

Format:(20A4)

Number of cards: 1

Procedure: Enter the title of the problem in columns 1-20.

Category 2:

Format: (2I10)

Number of cards: Equal to the number of nodes in the system.

Procedure: Enter the number of nodes in columns 1-10, and the number of elements in 11-20. Both integers must be right justified.

Category 3:

Format: (I10,3F10.0)

Number of cards: Equal to the number of nodes in the system.

Procedure: Each card contains the node number followed by the Z coordinate, R coordinate, and nodal blockage factor. The coordinates are in dimensions of inches.

Category 4:

Format: (9I5)

Number of cards: Equal to the number of elements in the system.

Procedure: Each card contains nine integers right justified in columns 5, 10, 15, etc., through 45. The first integer is the element number followed by the eight nodes associated with that element. It is important to note that the nodes are read in starting with the upper right hand node and proceeding in a counterclockwise fashion around the element.

Category 5:

Format: (2I10)

Number of cards: Equal to the number of

elements.

Procedure: Enter the element number in columns 1-10, followed by the integer '1' (duct), '2' (rotor), or '3' (stator) describing the element as either in a duct, rotor, or stator region.

Category 6:

Format: (6X,A4,I10,F10.0)

Number of cards: Equal to the number of rotor and stator nodes plus one 'STOP' card.

Procedure: Enter the node number (right justified) in columns 11-20 followed by the value of the associated absolute flow angle in radians in columns 21-30. The last card in this category is a 'STOP' card entered in columns 7-10.

Category 7:

Format: (6X,A4,I10,F10.0)

Number of cards: Equal to the number of rotor nodes plus one 'STOP' card.

Procedure: Enter the node number (right justified) in columns 11-20 followed by the value of the associated relative flow angle in radians in columns 21-30. The last card in this category is a 'STOP' card.

Category 8:

Format: (7F10.0),(F10.0)

Number of cards: 2

Procedure: Enter the following quantities in the prescribed order and with the noted dimensions.

First card

Mass flow rate: (lbm/sec)

Inlet axial velocity: (ft/sec)

Outlet axial velocity: (ft/sec)

Inlet total density: (lbm/ft^3)

Inlet static density: (lbm/ft^3)

Inlet total pressure: (lbf/in^2)

Inlet total temperature: (°R)

Second card

Speed: (RPM)

Category 9:

Format: (3F10.0)

Number of cards: 1

Procedure: Enter the following quantities in the prescribed order.

Gas constant: (ft-lbf/lbm-°R)

Ratio of specific heats

Constant pressure specific heat: (BTU/lbm-$^{\circ}$R)

Category 10:

Format: (F10.0)

Number of cards: 1

Procedure: Enter the first estimate of the internal stream functiion to be used in the first iteration.

Category 11:

Format: (6X,A4,I10,F10.0)

Number of cards: Equal to the number of nodes having a specified value of the stream function plus a 'STOP' card.

Procedure: This set of cards allows the stream function boundary conditions to be read in. A typical card contains an integer, right justified in columns 11-20 , which is the node number, followed by the value of the specified stream function in columns 21-30. The last card is a 'STOP' card.

Category 12:

Format: (6X,A4,I10)

Number of cards: Equal to the number of nodes where the right hand side is to be specified.

Procedure: Enter the node number, right justified in columns 11-20, where the right hand side is to be calculated. Again, the last card in this category is a 'STOP' card.

After all the data has been read by the program, the input data is printed and the mesh is plotted for verification by the user. The sample format is shown in Appendix C.

This concludes the input routine. The next section describes the calculation of the stiffness matrix, K.

2. Stiffness matrix evaluation

As shown previously in Section II.C.1, the following equation describes each term in the eight by eight elemental matrix.

$$K_{ij} = \int_{-1}^{1}\int_{-1}^{1} k \left\lfloor \frac{\partial N_i}{\partial \xi} \; \frac{\partial N_i}{\partial \eta} \right\rfloor \left\{ [J]^{-1} \right\}^{T} \left\{ \right\} \det [J] \, d\xi \, d\eta \qquad \text{(III.B.2.1)}$$

In addition, 'k' is defined in the following way in order to numerically integrate the equation.

$$K = \frac{1}{\sum_{i=1}^{8} p_i N_i(\xi,\eta) \cdot \sum_{i=1}^{8} r_i N_i(\xi,\eta) \cdot \bar{b}} \qquad \text{(III.B.2.2)}$$

where b is defined as the elemental blockage factor taken as an average over the eight nodes of the particular element and (ξ,η) are the defined Gauss-Quadrature integration points.

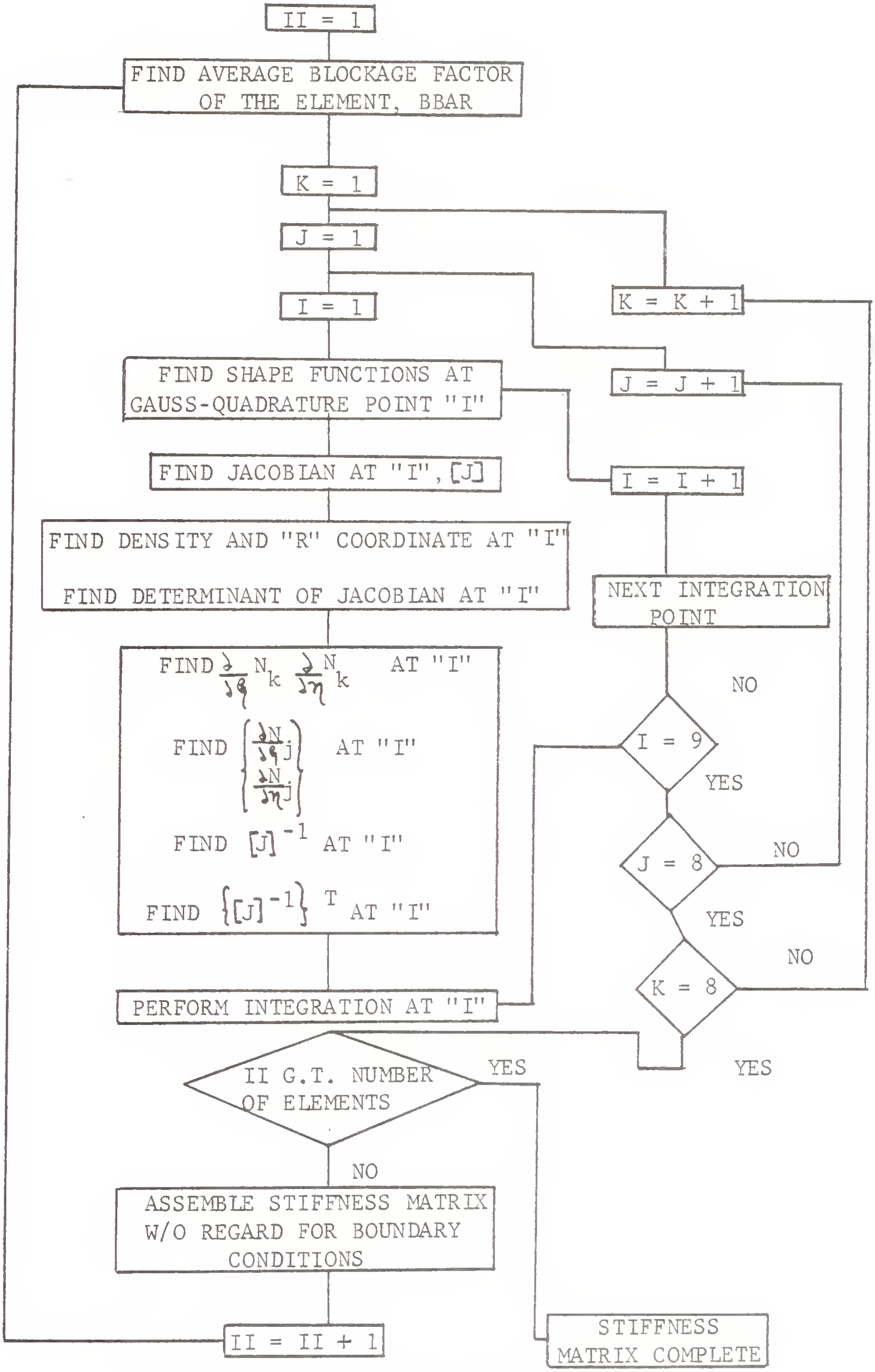


Figure 9 - STIFFNESS MATRIX EVALUATION

Fig 9 depicts the flowchart for both elemental stiffness matrix evaluation and the assemblage into the system stiffness matrix. More specifically, the figure shows a three-point Gaussian Quadrature scheme but can be changed to a two-point scheme by simply integrating four times instead of nine as shown.

The actual coding of the stiffness matrix evaluation and assemblage may be found in lines STR03510 through STR04770 in the computer program.

3. Solution of systems of equations

At this point, the system of equations are modified for the boundary conditions and solved for the nodal stream function values. An equation solving routine, DSIMQ, available in the system library was used for this purpose. It was found that no comparable savings was realised by using a banded equation solver.

4. Iteration schemes

As noted previously in Section II.C, a relaxation scheme is necessary for convergence to a solution.

Two distinct differences with regard to the iteration method were noted from that of Ref.7. Firstly, it was found that relaxation was necessary only in the rotor and stator elements and also in the duct region between the rotor outlet and stator inlet. Secondly, due to the extreme non linearity in the rotor-stator areas, a switch was required which changed the sign of α in equation (II.C.2.4) as required for stability of convergence. Clarification of

this change follows: It was found that during the initial three or four iterations, the stream function values of the rotor-stator nodes sometimes exceeded the value of the upper boundary. Due to an abscence of sources within the domain of solution, this occurrence was incompatible with the boundary conditions. At this point, it was necessary to make α negative in equation (II.C.2.4). During subsequent iterations, as the solution converged, the rotor-stator regions became stable and the sign of α was returned to its positive value. This iteration proved to stabilize the solution with respect to stream function values and velocities.

The iteration procedure is coded in the computer program from lines STR05070 through STR05200.

5. <u>The output routine</u>

Once convergence is obtained or the number of iterations have reached the limit imposed by the user, the results are displayed. A sample output is shown in the Appendix. In addition, the units of all dependent variables are the same as those noted in the input routine.

C. THE SUBROUTINES

The following describes each of the six subroutines in the computer program. Each subsection contains a list of calling arguements and for subroutines FCAL, SLINE, and VEL, a basic flowchart. In addition, for those subroutines whose mathematical theory was not presented in Section III, a brief treatment is also given.

1. Subroutine shape

This subroutine calculates the shape functions (equation (II.B.17)) at the values of ξ and η as requested in the arguement list below.

SUBROUTINE SHAPE (E,Z,SF)

E = value of ξ (input)

Z = value of η (input)

SF = eight by one vector of the eight shape functions.

2. Subroutine jacob

JACOB calculates the Jacobian matrix as defined in equation(II.C.1.4) for the value of ξ ,η denoted in the arguement list.

SUBROUTINE JACOB (E1,Z1,D,E,RC$,ZC$,RJAC)

E1 = value of η (input)

Z1 = value of ξ (input)

D = eight by one vector of $\frac{\partial N_i}{\partial \xi}$ (calculated)

E = eight by one vector of $\frac{\partial N_i}{\partial \eta}$ (calculated)

RC$ = eight by one vector of the 'r' coordinates of the nodes associated with the element (input)

ZC$ = eight by one vector of the 'z' coordinates of the nodes associated with the element (input)

RJAC = two by two Jacobian matrix (output)

In addition, the subroutine assumes that the vectors RC$ and ZC$ contain element coordinates arranged in a counter clockwise fashion beginning with the upper right corner node.

3. Subroutine sline

This subroutine calculates the thermodynamic variables throughout the machine given the inlet conditions as described in Section II.C.2. The calling arguements are defined below.

SUBROUTINE SLINE(UINLET,RC,PSI,WRL,H,UVEL,VVEL,TVEL, NODE,NNODEI,CP,TT,KK,ALP,WG,TWEL,BE,HS)

UINLET = Inlet axial velocity

RC = Nodal 'r' coordinates vector

PSI = Nodal stream function vector

WRL = Nodal angular momentum vector

H = Nodal total enthalpy vector

UVEL = Nodal axial velocity vector

VVEL = Nodal radial velocity vector

TVEL = Nodal absolute tangential velocity vector

NODE = Matrix containing nodes associated with the element

INLET = Vector containing node numbers at inlet station

NNODEI = Number of nodes at inlet station

CP = Specific heat

TT = Total temperature at inlet

KK = Iteration counter

NTE = Element type vector

ALP = Nodal absolute flow angle vector

TWEL = Nodal relative tangential velocity vector

BE = Nodal relative flow angle vector

HS = Nodal static enthalpy vector

As shown in Fig 10, the basic calculation procedure begins with calculating the required energy and momentum values at the inlet station. At this point, beginning with element one, the element type is interrogated to distinguish between duct, rotor, and stator elements. If the element is in a duct region, then the streamline intersections for local nodes 2,6,7,8 and 1 (Fig 7) are determined along with the associated values of energy and angular momentum. For the rotor and stator elements, one must initially find the energy and momentum values at local nodes 3,4,5 (Fig 7) due to the discontinuities imposed by the blade edges. Once these calculations are performed, then the process for the remaining nodes in the element proceeds in a similar fashion to the duct elements.

After all the elements have been cycled through, the new distributions of nodal angular momentum and energy are returned to the main program for further computations. Specifically, these values will be used by the next subroutine , FCAL, for calculation of the right hand side vector.

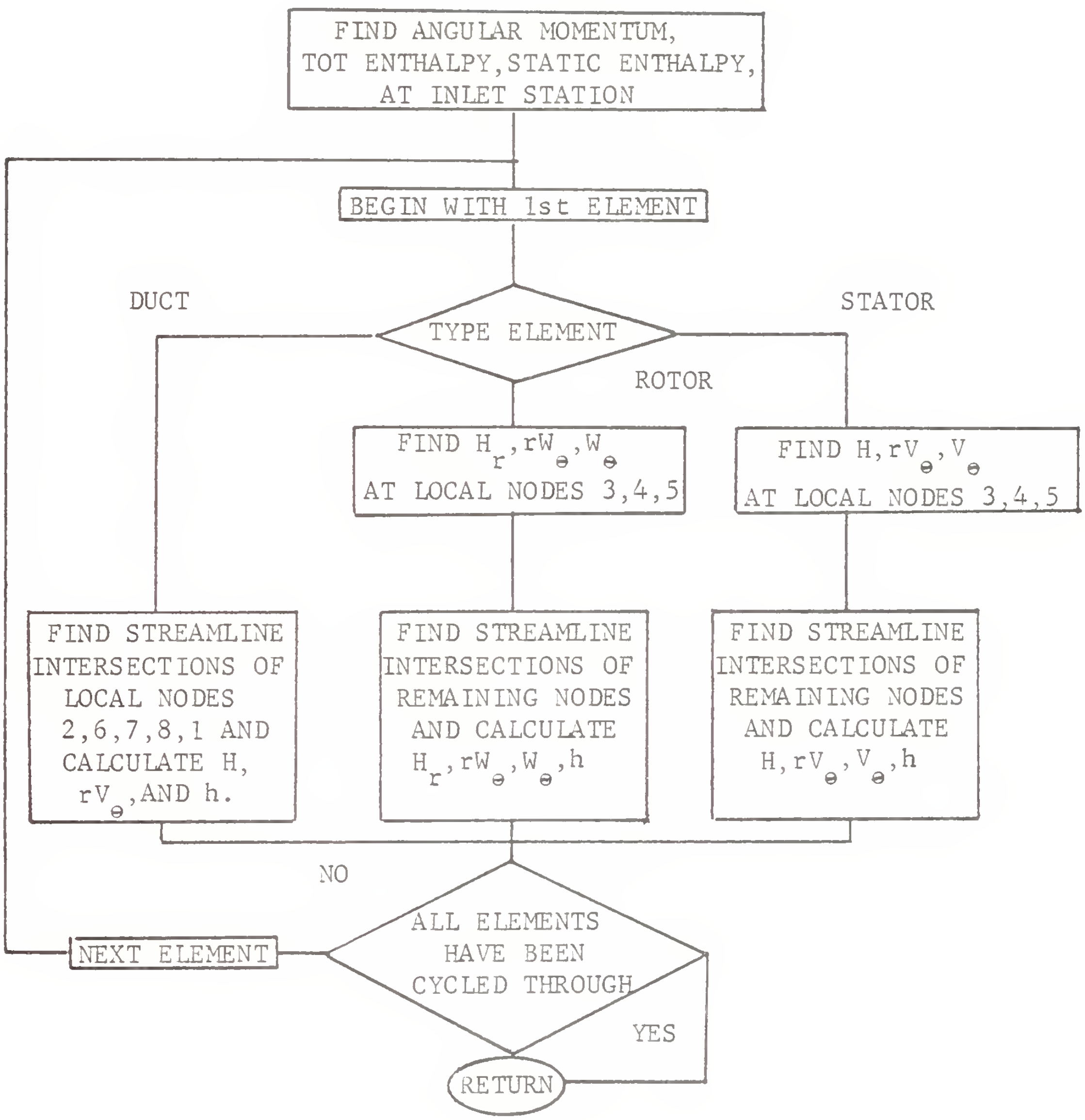


Figure 10 - SUBROUTINE SLINE

4. Subroutine fcal

FCAL calculates the right hand side vector as defined by equations (II.A.31) and (II.B.21). Using the identical coordinate transformations for numerical integration as described in Section II.C, the final equation to be coded is the following,

$$f_i = \int_{-1}^{1}\int_{-1}^{1} \frac{N_i}{\sum N_i V_{z_i}} \cdot \frac{\sum N_i V_{\theta i}}{\sum N_i R_i} \left\{ \left(\left[J^{-1}(2,1) \cdot \frac{\partial N_i}{\partial \xi} \right.\right.\right. \tag{III.C.4.1}$$

$$\left.\left.\left. + J(2,2) \cdot \frac{\partial N_i}{\partial \eta} \right] \right) (W_i - H_i) \right\} \det J \, d\xi \, d\eta$$

where isentropic flow is assumed, and,

W_i = angular momentum

(III.C.4.2)

H_i = total enthalpy

The arguement list is defined below. In addition, only those variables in the list which have not been defined previously are described.

SUBROUTINE FCAL(F,W,H,ZA,EA,UVEL,RC,ZC,WRL,TVEL,NFS,NODE,NN,NE,NNFSP,TWEL,NFE)

F = Right hand side vector, f(r,z)

W = vector of gaussian quadrature coefficients

ZA = Vector of ξ_i gaussian quadrature points

EA = Vector of η_i gaussian quadrature points

NFS = Vector containing nodes where the right hand side

is to be specified

NN = number of nodes

NE = number of elements

NNFSP = Number of nodes where the right hand side is specified

Fig 11 depicts the basic flowchart for the subroutine. To initialize the procedure, one begins with the first node (upper right hand corner) of the first element. A switch is then applied which determines if the right hand side is to be calculated at the node or if a stream function value has been specified. This information is transferred from the main program through the arguement list. Once the node is allowed through the switch, then the integration process is started at the first integration point. As in Section III.A.2, the flowchart depicts a three-point Gauss Quadrature scheme. After the integration has been completed, a switch determines if all the local nodes in the element have been cycled through and if so, then the assembly of the elemental vector, F$, is performed to build the system right hand side vector , F. Finally, the subroutine determines if all the elements have been examined in order to signal completion of the right hand side vector. At this point, the vector, F(r,z), is returned to the main program for problem solution.

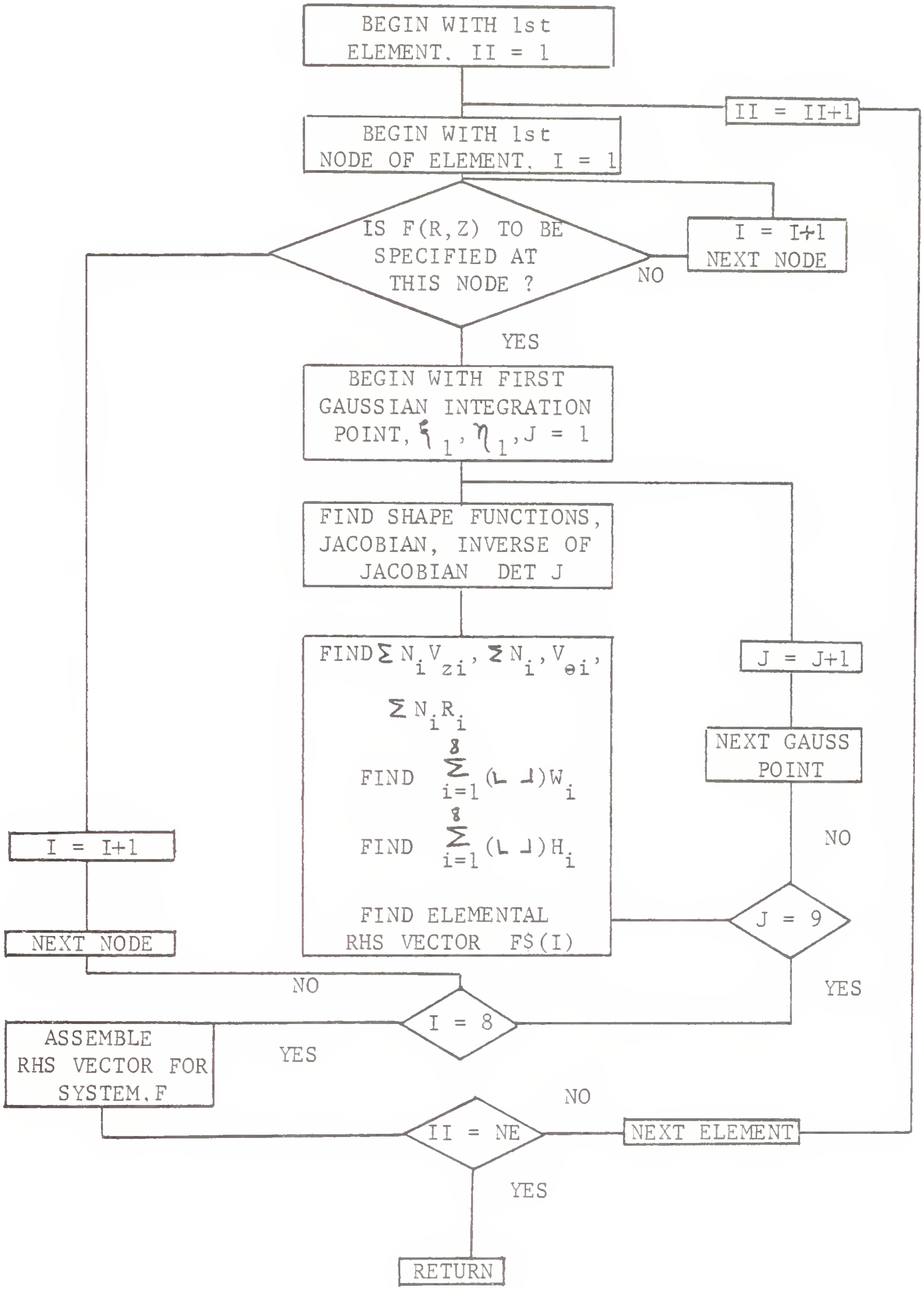


Figure 11 - SUBROUTINE FCAL

5. <u>Subroutine vel</u>

This subroutine calculates axial and radial velocities and also densities at each of the nodes from a known stream function distribution. As noted previously in Section II.C.2, both velocity and density profiles are updated after obtaining the latest value of nodal stream function.

The velocity calculation proceeds from the stream function equations,

$$V_z = \frac{1}{\rho r b} \frac{\partial \psi}{\partial r} \qquad \text{(III.C.5.1)}$$

$$V_r = -\frac{1}{\rho r b} \frac{\partial \psi}{\partial z} \qquad \text{(III.C.5.2)}$$

where 'b' is the tangential blockage factor. Since r, ,and are of the following form,

$$r = \sum_{i=1}^{8} r_i N_i$$

$$\rho = \sum_{i=1}^{8} \rho_i N_i \qquad \text{(III.C.5.3)}$$

$$\psi = \sum_{i=1}^{8} N_i \psi_i$$

then the equation for the axial velocity, V_z, becomes,

$$V_z = \frac{1}{b \sum_{i=1}^{8} \rho_i N_i \sum_{i=1}^{8} N_i r_i} \left[\sum_{i=1}^{8} \frac{\partial N_i}{\partial r} \psi_i \right] \qquad \text{(III.C.5.4)}$$

Again, since the shape function, $N_i(\xi,\eta)$, is not an

implicit function of 'r' and 'z', one must use equation (II.C.1.5) to obtain the proper derivatives for computation of equation (III.C.5.4). For example, from equation (III.C.5),

$$\sum_{i=1}^{8} \frac{\partial N_i}{\partial r} = \sum_{i=1}^{8} \left[J^{-1}(1,1) \cdot \frac{\partial N_i}{\partial \xi} + J^{-1}(1,2) \frac{\partial N_i}{\partial \eta} \right] \qquad \text{(III.C.5.5)}$$

At this point, with equation (III.C.5.5) substituted into equation (III.C.5.4), one has the complete expression for the axial velocity as functions of ξ, η. One proceeds similarly for expressing the radial velocity, V, in terms of ξ and η.

In order to calculate the nodal density, one uses the following density relation for flows in the stator and duct regions.

$$\frac{\rho}{\rho_t} = \left(1 - \frac{\gamma - 1}{2a_0} V^2\right)^{\frac{1}{\gamma - 1}} \qquad \text{(III.C.5.6)}$$

where ρ_t is the stagnation density.

Since,

$$V^2 = (V_z^2 + V_r^2)(1 + \tan^2 \alpha) \qquad \text{(III.C.5.7)}$$

then,

$$\frac{\rho}{\rho_t} = \left[1 - \frac{\gamma - 1}{2a_0^2} \left(\frac{1}{\rho r b}\right)^2 (\psi_r^2 + \psi_z^2)(1 + \tan^2 \alpha)\right]^{\frac{1}{\gamma - 1}} \qquad \text{(III.C.5.8)}$$

Since the density appears on both sides of the equation, the new nodal density is obtained iteratively at the node.

For the relative flows in the rotor, the following relation for static density is used [Ref.14].

$$\frac{\rho}{\rho_t} = \left[1 - (\gamma-1)\frac{\omega R V_\theta}{a_o^2} - \frac{(\gamma-1)}{2}\,\frac{W^2 - \omega^2 R^2}{a_o^2}\right]^{\frac{1}{\gamma-1}} \qquad \text{(III.C.5.9)}$$

Again, the solution of the nodal density is obtained in an iterative fashion.

In the following arguement list, only those variables not defined in the previous subroutine descriptions are noted.

SUBROUTINE VEL(NE,NN,RC,NODE,G,RG,TT,RHOT,RHON,ZC, PSI,RHO,B,UINLET,UVEL,VVEL,RHOSTA,NTE,ALP)

G = Ratio of specific heats

RG = Gas constant

RHOT = Total density at the inlet

RHON = Work vector which contains the new nodal density distribution

RHO = Nodal static density vector

B = nodal blockage factor vector

RHOSTA = Static density at the inlet station

The basic flowchart for SUBROUTINE VEL is shown in Fig 12. Beginning with the first node of the first element, the Jacobian matrix (equation (II.C.1.4)) and its inverse

are found. At this point the partial derivatives with respect to 'r' and 'z' of the shape functions are found as noted in equation (III.C.5.5). A switch then allows those nodes not at the inlet station to pass and calculates the new density and velocities at the nodes. For those nodes at the inlet, the velocities and static densities are retained at the given inlet conditions. This is done to maintain boundary condition integrity for the solution. After cycling through all elements, the subroutine returns the new nodal velocity and density distributions to the main program.

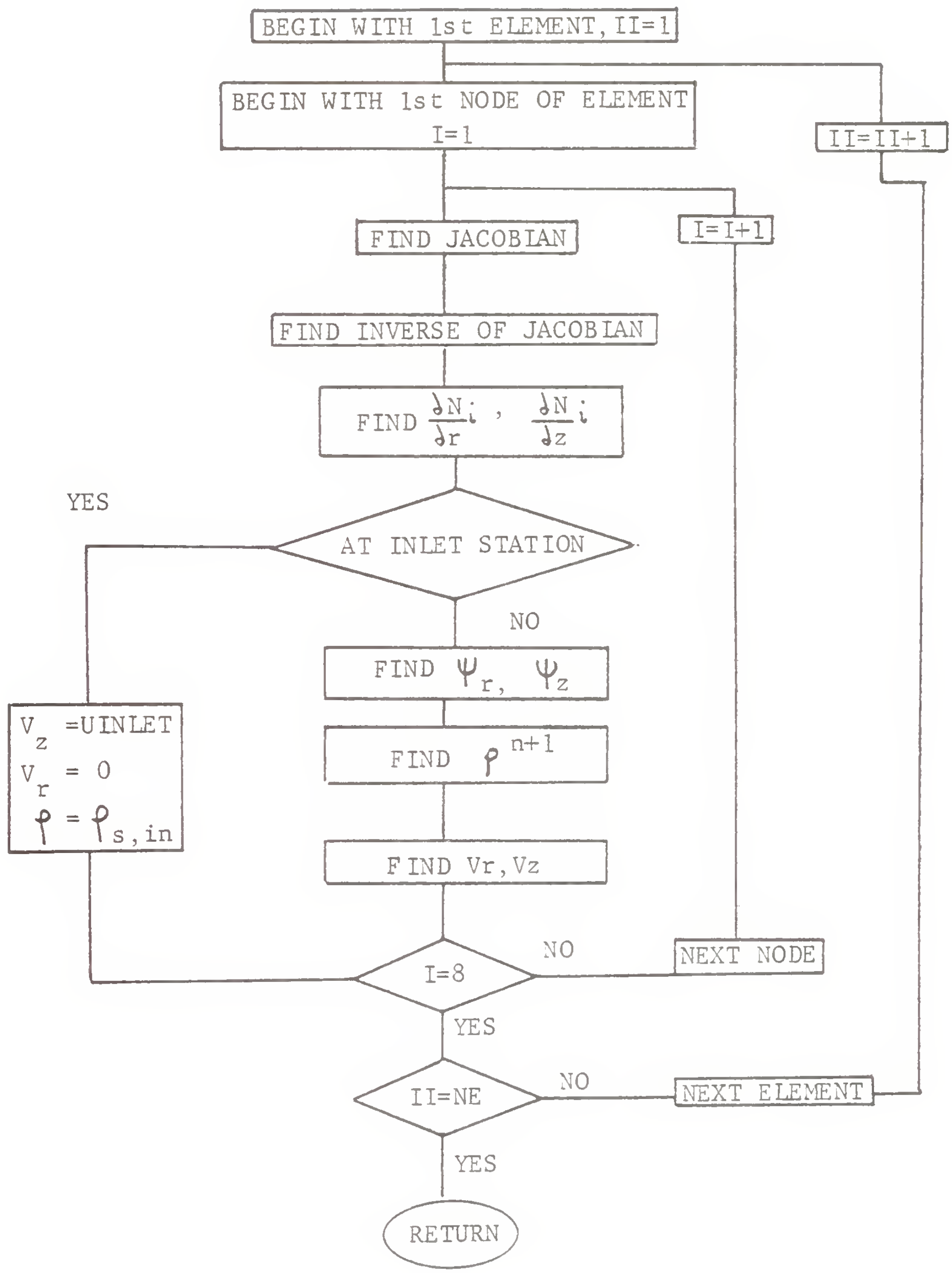


Figure 12 - SUBROUTINE VEL

6. Subroutine mplot

This subroutine utilizes the Calcomp plotter to depict the mesh topology of the machine under observation.

```
SUBROUTINE MPLOT(RC,ZC,NODE,NN,NE)
```

This completes the description of the main program and associated subroutines. In the next section, a test case is carried through from data input to final results.

IV. TEST CASES AND RESULTS

The program was tested by using published performance data [Ref.12] of the NASA Task-1 stage transonic compressor. The compressor was discretized into twenty-eight elements and 107 nodes with 15 axial calculation stations (Fig 6). The speed was 0.5 design speed with a mass flow of 107.6 lbm/sec. In addition, uniform flow was assumed both at the inlet and outlet stations. Turning angles for the rotor and stator were pre calculated assuming uniform conditions at the rotor inlet and using NASA SP-36 blade correlation data [Ref.13]. These absolute and relative flow angles were assumed constant throughout the iterative procedure as they were an integral part of the input data. The Appendix contains a listing of the input data and output results for the NASA Task-1 transonic compressor with test conditions noted. To compare the accuracy of the predicted flow with actual laboratory observations, computed axial velocity profiles at the rotor inlet, rotor outlet, stator inlet, and stator outlet were compared with experimental results. In addition numerical results from Ref.7 were also compared.

Fig 13-16 show the computer predictions plotted with the experimental values and the numerical solutions obtained by Hirsch and Warzee. The profiles shown were obtained after ten iterations and using a relaxation factor of 0.2. The figures show that the best overall agreement with experimental data occurred in the stator inlet and outlet. In this region the worst error was 17% which occurred at the stator tip inlet. The average error throughout the stator region with respect to experimental data was 6.6%.

The rotor hub and tip outlet area exhibited instabilities in density convergence using equation III.C.5.9. Specifically, the density solution converged to within 8% at the rotor outlet tip and hub. It was found that by not allowing the nodal density at these nodes to go below a critical value of 0.06 lbm/cu ft, the solution for the stream function converged. By allowing the nodal densities at the rotor outlet tip and hub to go below this critical value, the computed velocities at these nodes became increasingly large and the arguement within the brackets of equation III.C.5.9 became less than one. This prevented continuation of the iterations for the stream function solution. In addition, the rotor tip outlet exhibited more instability than the rotor hub outlet. The static density at the rotor hub outlet oscillated about a value of 0.062 lbm/cu ft while the rotor tip outlet was constantly driven to the critical value of 0.06 lbm/cu ft. One method attempted to alleviate this problem was the following. Since a half-interval iteration routine was used, one trial run involved reversing the direction of consecutive guesses when the density iteration did not converge. It was found however, that after three to four iterations of the system of equations, the static densities at the rotor outlet tip and hub were again driven to smaller and smaller values which led to instability once more. The nodal densities converged at all interior points of the rotor edge and mid-blade regions and also at all the rotor inlet nodes. By including all rotor nodes, the average error with respect to experimental data was 27.5%.

Fig 17 shows a plot of convergence criteria, ϵ, versus the number of iterations for a relaxation factor of 0.2. The stability of convergence is shown to initially decrease and then after the third iteration oscillates about an approximate value of 28%. It is important to note that this curve represents the maximum value of ϵ as shown in equation

(II.C.2.5). In addition, the curve in actuality represents the oscillation of nodal stream function values in the rotor/stator regions since in fact this is where the non-linearity is the greatest.

V. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

Agreement with both experimental data and numerical solution of Ref.7 was best in the stator region to within 8%. Predicted axial velocity profiles in the rotor inlet area were within 26.2% of experimental results. The instabilities with respect to static density solutions are prevalent. One of the reasons for this numerical disagreement with Hirsch and Warzee is the isentropic assumption imposed by the present program. Recommendations for further study on the project include the addition of entropy variations in the rotor and stator blade regions. This would necessitate the use of blade correlation data [Ref.13] for loss predictions and involve additional input data plus program additions to Subroutine's SLINE and FCAL.

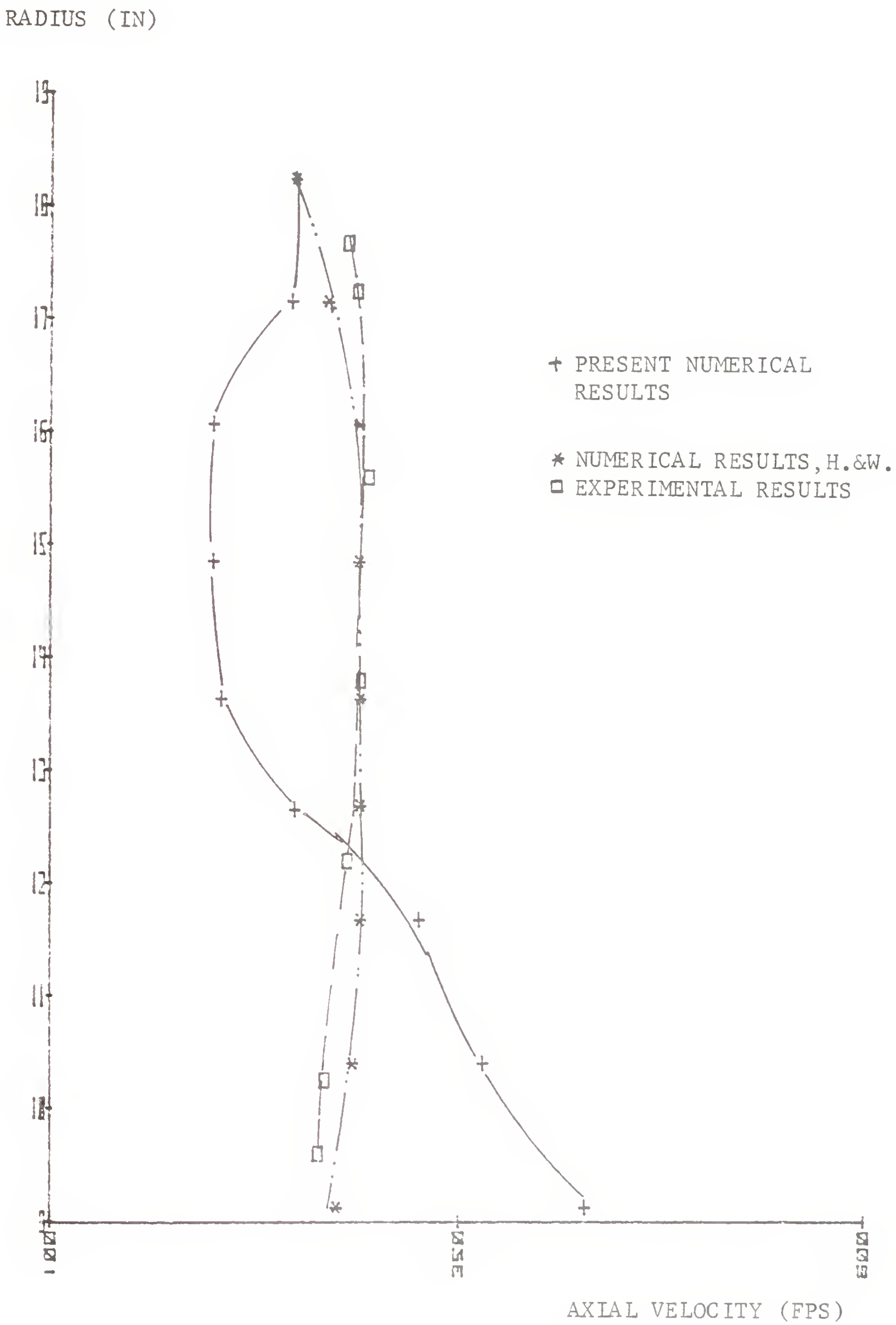


Figure 13 - AXIAL PROFILE AT ROTOR INLET

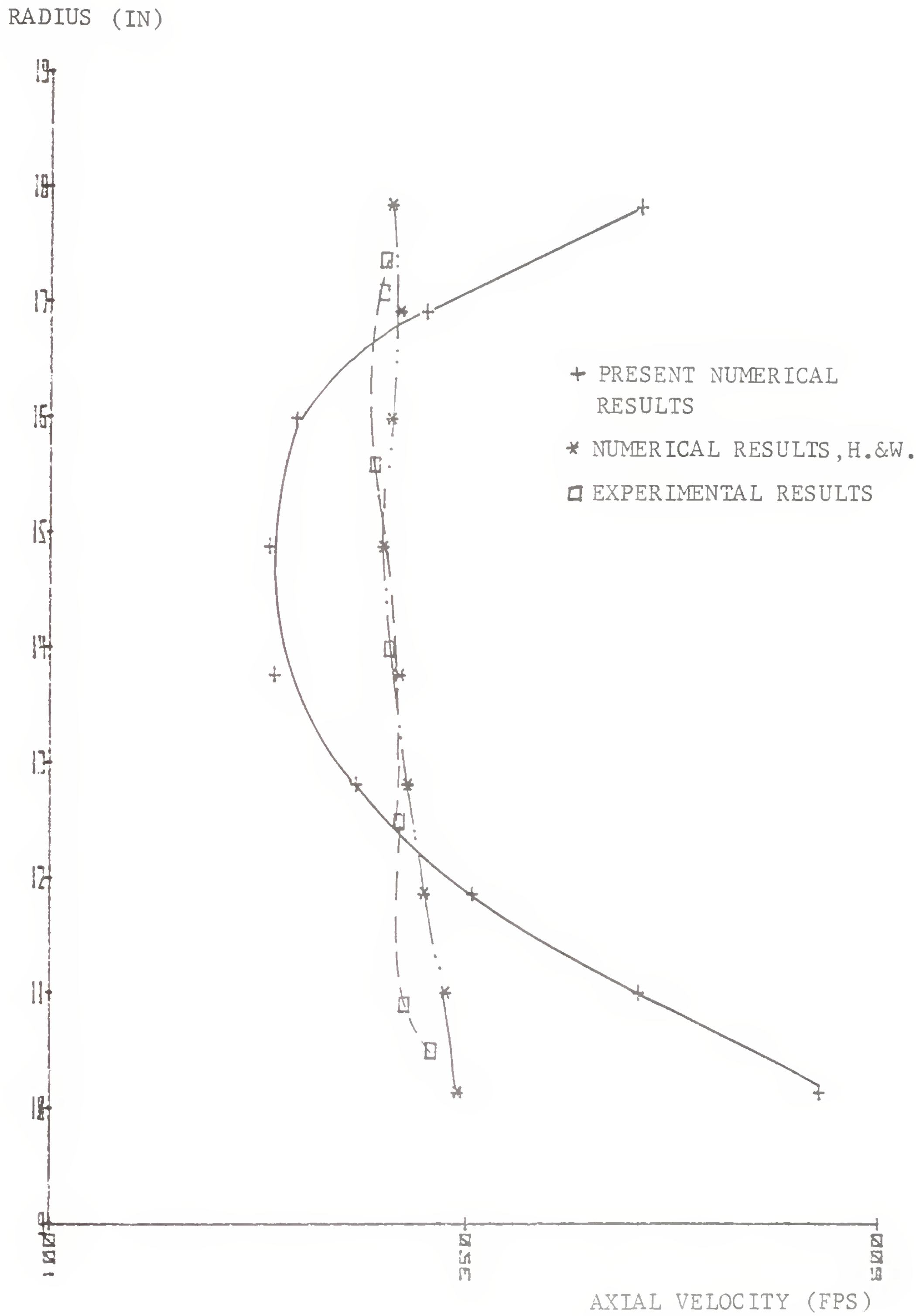


Figure 14 - AXIAL PROFILE AT ROTOR OUTLET

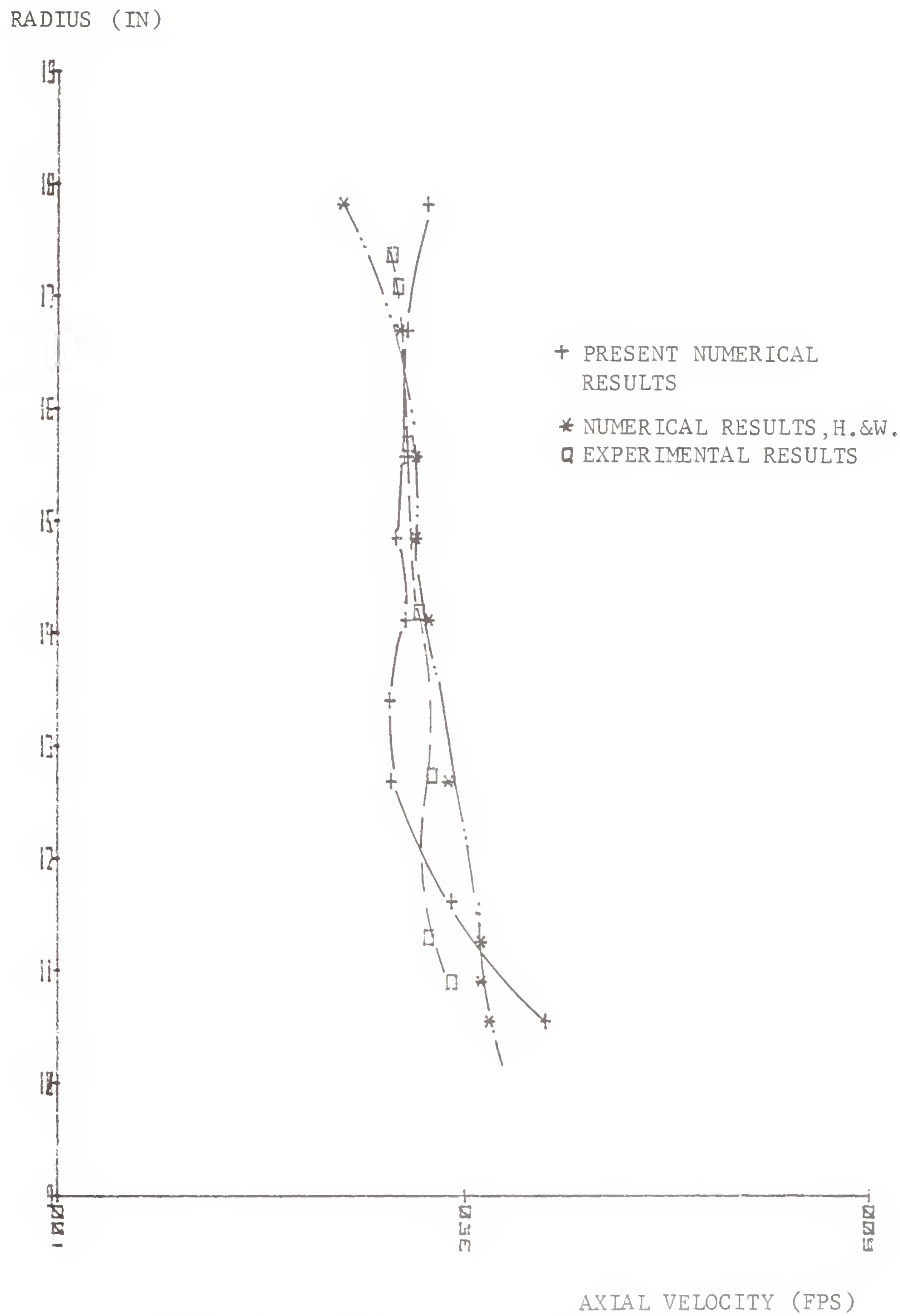


Figure 15 - AXIAL PROFILE AT STATOR INLET

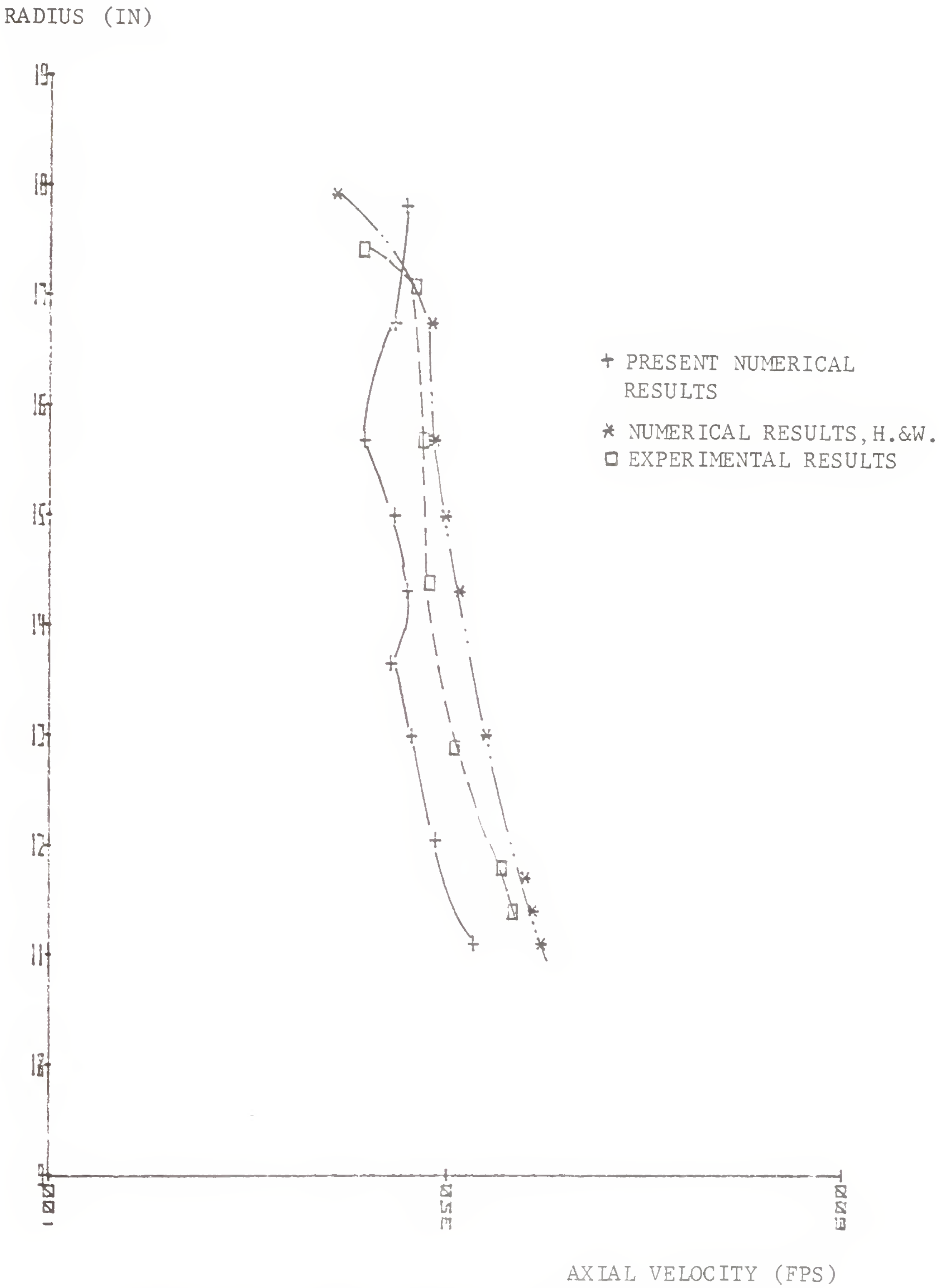


Figure 16 - AXIAL PROFILE AT STATOR OUTLET

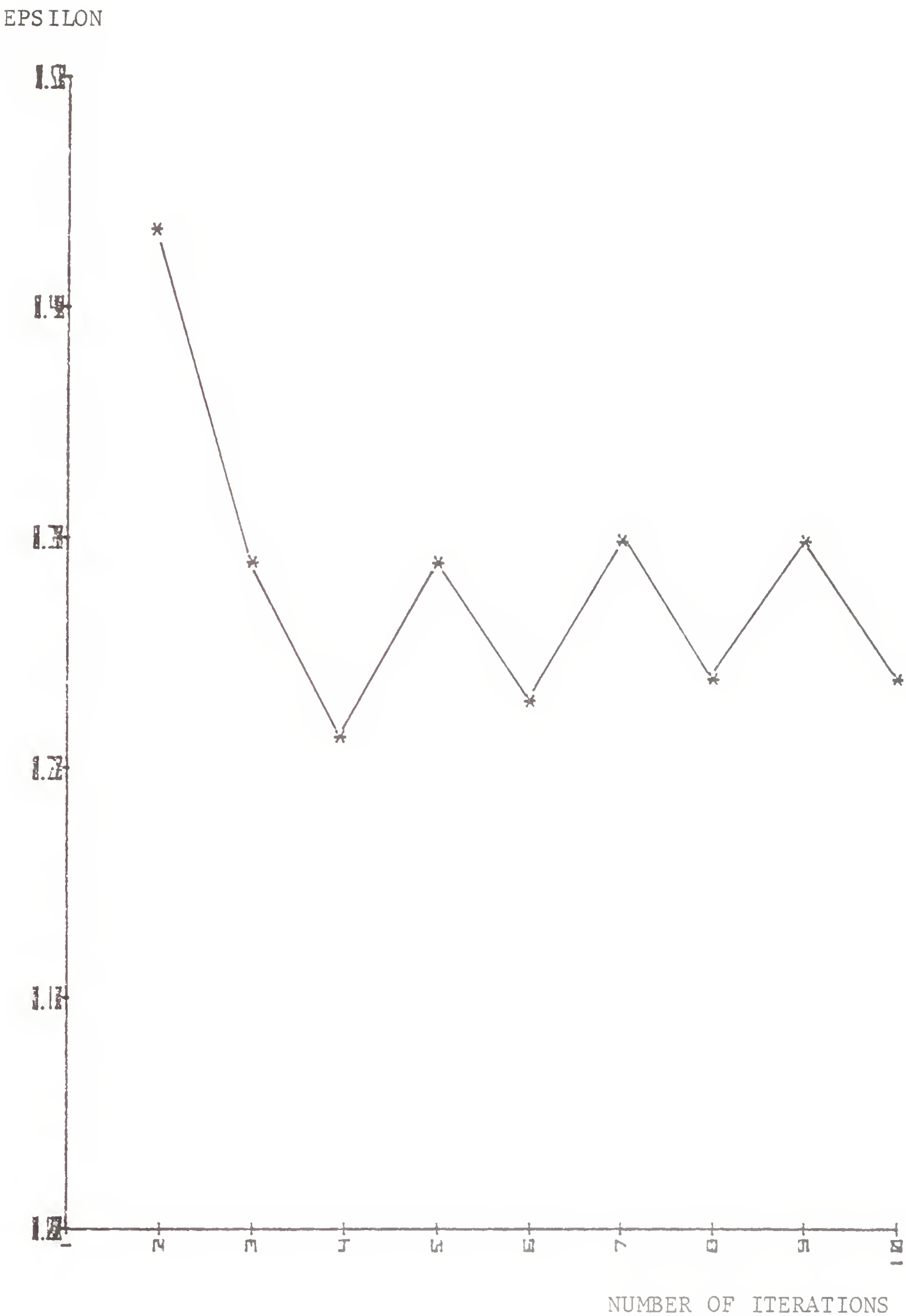


Figure 17 - EPSILON VS. ITERATIONS

APPENDIX A
COMPUTER PROGRAM

```
C
C               THE FINITE ELEMENT METHOD APPLIED TO TWO-DIMENSIONAL.
C               AXI-SYMMETRIC,INVISCID,COMPRESSIBLE FLOW IN A TURBO-
C               MACHINE.  STREAM FUNCTION FORMULATION IS USED ALONG WITH
C               EIGHT-NODE ISOPARAMETRIC ELEMENTS.  TWO POINT OR THREE
C               POINT GAUSSIAN QUADRATURE FOR NUMERICAL INTEGRATIONS
C               ARE USED.
      IMPLICIT REAL*8(A-H,P-Z)
      DATA NREAD/4/
      DATA NWRITE/7/
      DATA STOP/'STOP'/
C
C               DIMENSION ALL ARRAY VARIABLES
C
      DIMENSION RJAC(2,2),ROW(1,2),COL(2,1),EM$(8,8),RIJAC(2,2),R(40)
      DIMENSION EM(107,107),F(107),PSI(107),RHS(107),TIJAC(2,2)
      DIMENSION PROD1(1,2),PROD2(1,2),PROD3(1),FNZ1(1),FNZ2(1),W(9)
      DIMENSION D(8),E(8),SF(8),ZA(9),EA(9),RHOE(40)
      DIMENSION UVEL(107),VVEL(107),PRES(107),RC$(8)
      DIMENSION ZC$(8),RC(107),ZC(107),RHO(107),B(107),RHON(107)
      DIMENSION WRL(107)
      DIMENSION TVEL(107),PSIO(107),H(107),ALP(107),TWEL(107),BE(107)
      DIMENSION RTEMP(107),HS(107)
      DIMENSION NPSIS(107),NFIS(107)
      DIMENSION NFS(107),NPSI(107),NODE(28,8),N(8),L1(2),M1(2),INLET(10)
      DIMENSION IULET(10),NTE(40),TITLE(10)
C
C               TWO-POINT GAUSSIAN POINTS.
C
      DATA ZA/.57735027D0,-.57735027D0,-.57735027D0,.57735027D0/
      DATA EA/.57735027D0,.57735027D0,-.57735027D0,-.57735027D0/
C
C               THREE-POINT GAUSSIAN POINTS.
C
      DATA ZA/.774596669242D0,.774596669242D0,.774596669242D0,
     10.D0,0.D0,0.D0,-.774596669242D0,-.774596669242D0,
     2-.774596669242D0/
      DATA EA/.774596669242D0,0.D0,-.774596669242D0,
     1.774596669242D0,0.D0,-.774596669242D0,
     2.774596669242D0,0.D0,-.774596669242D0/
C               WEIGHTING FUNCTIONS FOR 3 PT. GAUSSIAN QUAD. PTS.
C
      DATA W/.308641975D0,.4938271604D0,.308641975 2D0,
     1.4938271604D0,.790123456 6D0,.4938271604D0,
     2.3086419752D0,.4938271604D0,.3086419752D0/
C
C               READ IN TITLE
      READ(NREAD,50)TITLE
```

```
50    FORMAT(10A4)
C
C
C                READ IN NUMBER OF NODES AND NUMBER OF ELEMENTS
C
      READ(NREAD,100) NN,NE
100   FORMAT(2I10)
C     INITIALIZE ALL MATRICES
      DO 110 I =1,NN
      F(I) = 0.D0
      PSI(I) = 0.D0
      RHS(I) = 0.D0
      NPSIS(I) = 0
      UVEL(I) = 0.D0
      VVEL(I) = 0.D0
      TVEL(I) = 0.D0
      PRES(I) = 0.D0
      RC(I) = 0.D0
      ZC(I) = 0.D0
      B(I) = 0.D0
      NFS(I) = 0
      NPSI(I) = 0
      WRL(I) = 0.D0
      RHO(I) = 0.D0
      RHON(I) = 0.D0
      H(I) = 0.D0
      HS(I) = 0.D0
      ALP(I) = 0.D0
      TWEL(I) = 0.D0
      BE(I) = 0.D0
110   CONTINUE
C                ZEROIZE OUT INLET( ) AND IULET( )
      DO 111 I = 1,10
      INLET(I) = 0
      IULET(I) = 0
111   CONTINUE
      DO 140 I = 1,2
      ROW(1,I) = 0.D0
      COL(I,1) = 0.D0
      PROD1(1,I) = 0.D0
      PROD2(1,I) = 0.D0
140   CONTINUE
      DO 150 I = 1,NN
      DO 150 J = 1,NN
      EM(I,J) = 0.D0
150   CONTINUE
      DO 160 I = 1,8
      DO 160 J = 1,8
```

```
      EM$(I,J) = 0.D0                                                   STR00970
160   CONTINUE                                                          STR00980
      DO 165 I = 1,NE                                                   STR00990
      NTE(I) = 0                                                        STR01000
      RHOE(I) = 0.D0                                                    STR01010
      R(I) = 0.D0                                                       STR01020
      DO 165 J = 1,8                                                    STR01030
      NODE(I,J) = 0                                                     STR01040
165   CONTINUE                                                          STR01050
C               READ NODE NUMBERS,NODAL COORDINATES (IN INCHES), AND NODSTR01060
C               AL BLOCKAGE FACTOR. INLET STATION ZC() MUST BE = 0.D0.  STR01070
C               LAST NODAL ZC(NN) MUST BE AT THE OUTLET STATION.        STR01080
      DO 170 J = 1,NN                                                   STR01090
      READ(NREAD,1000)I,ZC(I),RC(I),B(I)                                STR01100
1000  FORMAT(I10,3F10.0)                                                STR01110
170   CONTINUE                                                          STR01120
C                                                                       STR01130
C     READ IN SYSTEM TOPOLOGY,I.E.,ELEMENT NO. AND LOCAL NODE NUMBERS.  STR01140
C     STARTING AT UPPER RIGHT HAND CORNER AND TRAVERSING IN A CCW       STR01150
C     FASHION AROUND THE QUADRILATERAL.                                 STR01160
C                                                                       STR01170
      DO 180 L = 1,NE                                                   STR01180
      READ(NREAD,1010) J,NODE(J,1),NODE(J,2),NODE(J,3),NODE(J,4)        STR01190
     1,NODE(J,5),NODE(J,6),NODE(J,7),NODE(J,8)                          STR01200
1010  FORMAT (9I5)                                                      STR01210
180   CONTINUE                                                          STR01220
C                                                                       STR01230
C               READ IN TYPE OF ELEMENT:  NTE()=1(DUCT),NTE()=2(ROTOR)  STR01240
C               NTE()=3(STATOR)                                         STR01250
      DO 185 I = 1,NE                                                   STR01260
      READ(NREAD,1011)J,NTE(J)                                          STR01270
1011  FORMAT(2I10)                                                      STR01280
185   CONTINUE                                                          STR01290
C                                                                       STR01300
C               READ IN STATOR/ROTOR NODAL ABS FLOW ANGLES              STR01310
C               REMEMBER THAT THIS ASSUMES UNIFORM FLOW CONDITIONS AT   STR01320
C               ROTOR INLET.                                            STR01330
C                                                                       STR01340
      DO 186 I = 1,NN                                                   STR01350
      READ(NREAD,1012)WORD,J,ANGLE                                      STR01360
1012  FORMAT(6X,A4,I10,F10.0)                                           STR01370
      IF(WORD.EQ.STOP)GOTO1014                                          STR01380
      ALP(J) = ANGLE                                                    STR01390
186   CONTINUE                                                          STR01400
C                                                                       STR01410
C                                                                       STR01420
C               READ IN ROTOR NODAL REL FLOW ANGLES.                    STR01430
C                                                                       STR01440
```

```
1014  DO 187 I = 1,NN                                                        STR01450
      READ(NREAD,1013)WORD,J,ANGLE                                      STR01460
 1013 FORMAT(6X,A4,I10,F10.0)                                             STR01470
      IF(WORD.EQ.STOP)GOTO1034                                          STR01480
      BE(J) = ANGLE                                                     STR01490
  187 CONTINUE                                                          STR01500
C                                                                       STR01510
C                                                                       STR01520
C           READ IN INLET THERMODYNAMIC QUANTITIES; FLOW RATE,INLET STR01530
C           U VELOCITY,OUTLET U VELOCITY,RHOT,RHOSTA,PTOT,TTOT,N    STR01540
C           UNITS ARE AS FOLLOWS;  FLOW RATE (LBM/SEC)             STR01550
C           VELOCITY (FT/SEC) ; RHOT AND RHOSTA (LBM/CU FT) ;      STR01560
C           PTOT (PSF) ; TT ( DEGREES RANKINE) ; SPEED (RPM)      STR01570
C                                                                       STR01580
1034  READ(NREAD,1035)WDOT,UINLET,UOULET,RHOT,RHOSTA,PT,TT              STR01590
1035  FORMAT(7F10.0)                                                    STR01600
      READ(NREAD,1037)SPEED                                             STR01610
1037  FORMAT(F10.0)                                                     STR01620
C                                                                       STR01630
C           FIND OMEGA (RAD/SEC)                                        STR01640
C                                                                       STR01650
      WG = SPEED*2.D0*3.141593D0/60.D0                                  STR01660
C                                                                       STR01670
C                                                                       STR01680
C           READ IN FLUID/GAS CONSTANTS:  I.E., R(GAS CONSTANT),    STR01690
C           GAMMA(RATIO OF SPECIFIC HEATS), AND CP (BTU/(LBM - DEGRESTR01700
C           F RANKINE))                                                 STR01710
C                                                                       STR01720
      READ(NREAD,1036)RG,G,CP                                           STR01730
1036  FORMAT(3F10.0)                                                    STR01740
C                                                                       STR01750
C           READ IN FIRST ESTIMATE OF PSI DISTRIBUTION THROUGHOUT THSTR01760
C           E SYSTEM.                                                   STR01770
      READ(NREAD,1210)PSII                                              STR01780
1210  FORMAT(F10.0)                                                     STR01790
C                                                                       STR01800
C                                                                       STR01810
C           READ IN FIRST ESTIMATES OF UVEL(I) AND RHO(I).            STR01820
C                                                                       STR01830
      DO 166 I = 1,NN                                                   STR01840
      UVEL(I) = UINLET*12.D0                                            STR01850
      RHO(I) = RHOSTA                                                   STR01860
      PSI(I) = PSII                                                     STR01870
      PSIO(I) = PSI(I)                                                  STR01880
166   CONTINUE                                                          STR01890
C                                                                       STR01900
C           CALCJLATE INLET AND OUTLET PSI DISTRIBUTION BY FIRST    STR01910
C           FINDING THE NODES AT INLET AND OUTLET STATIONS. AFTER   STR01920
```

```
C                 FINDING THE NODES, PROCEED TO CALCULATE THE VALUES OF
C                 PSI BY ASSUMING UNIFORM FLOW CONDITIONS AT INLET AND
C                 OUTLET BY ASSUMING RHO = STATIC DENSITY AT INLET(RHOSTA)
C
C                 FIND NODES AT INLET STATION
      KIS = 0
      DO 231  I = 1,NN
      IF(ZC(I).NE.0.DO)GOTO231
      KIS = KIS + 1
      INLET(KIS) = I
      NNODEI = KIS
231   CONTINUE
C                 FIND NODES AT OUTLET STATION
      KIS = 0
      DO 232 I = 1,NN
      IF(ZC(I).NE.ZC(NN))GOTO232
      KIS = KIS + 1
      IULET(KIS) = I
      NNODEO = KIS
232   CONTINUE
C                 READ NODES WHERE PSI IS SPECIFIED
C
      DO 190 I = 1,NN
      READ(NREAD,1020)WORD,NPSP,PS
1020  FORMAT(6X,A4,I10,F10.0)
      IF(WORD.EQ.STOP)GOTO191
      NPSIS(I) = NPSP
      PSI(NPSIS(I)) = PS
      PSIO(NPSIS(I)) = PS
190   CONTINUE
C
C                 COUNT NODES HAVING SPECIFIED PSI
191   NNSPSI = I-1
C                 INSERT INLET PSI DISTRIBUTION FROM 'U' VEL DISTRIBUTION.
      DO 192 I = 1,NNODEI
      NPSIS(NNSPSI + I) = INLET(I)
      PSI(INLET(I)) = UINLET*(RC(INLET(I))**2 - RC(NNODEI)**2)/2.DO
      PSI(INLET(I)) = PSI(INLET(I))*RHO(INLET(I))*B(INLET(I))/144.DO
      PSIO(INLET(I)) = PSI(INLET(I))
192   CONTINUE
      NNSPSI = NNSPSI + NNODEI
C                 INSERT OUTLET PSI DISTRIBUTION FROM 'U' VEL DISTRIBUTION
      DO 193 I = 1,NNODEO
      NPSIS(NNSPSI + I) = IULET(I)
      PSI(IULET(I)) = UOULET*(RC(IULET(I))**2 - RC(NN)**2)/2.DO
      PSI(IULET(I)) = PSI(IULET(I))*RHO(IULET(I))*B(IULET(I))/144.DO
      PSIO(IULET(I)) = PSI(IULET(I))
193   CONTINUE
```

```
C                                                                       STR02410
C                RECOUNT NUMBER OF NODES WITH SPECIFIED PSI BY INCLUDING STR02420
C                THE INLET AND OUTLET NODES.                            STR02430
C                                                                       STR02440
      NNSPSI = NNSPSI + NNODEO                                          STR02450
C                READ NODES WHERE F(R,Z) IS SPECIFIED                   STR02460
      DO 200 I = 1,NN                                                   STR02470
      READ(NREAD,1030) WORD,NFSP                                        STR02480
1030  FORMAT(6X,A4,I10)                                                 STR02490
      IF(WORD.EQ.STOP)GOTO201                                           STR02500
      NFS(I) = NFSP                                                     STR02510
200   CONTINUE                                                          STR02520
201   NNFSP = I-1                                                       STR02530
C                                                                       STR02540
C                NFIS IS A LIST OF THE INDICES OF THE KNOWN F(R,Z)      STR02550
      DO 210 I = 1,NNFSP                                                STR02560
      NFIS(I) = NFS(I)                                                  STR02570
210   CONTINUE                                                          STR02580
C                                                                       STR02590
C                NPSI IS A LIST OF  THE INDICES IF THE KNOWN PSI        STR02600
      DO 220 I = 1,NNSPSI                                               STR02610
      NPSI(I) = NPSIS(I)                                                STR02620
220   CONTINUE                                                          STR02630
C                                                                       STR02640
C                NTOTF IS THE TOTAL NUMBER OF KNOWN F(R,Z)              STR02650
      NTOTF = NNFSP                                                     STR02660
C                                                                       STR02670
C                NTOTP IS THE TOTAL NUMBER OF KNOWN PSI                 STR02680
      NTOTP = NNSPSI                                                    STR02690
C                PRINT ALL INPUT DATA                                   STR02700
C                                                                       STR02710
C     GO TO 1101                                                        STR02720
221   WRITE(NWRITE,1038)TITLE                                           STR02730
1038  FORMAT(' ',20X,10A4)                                              STR02740
      WRITE(NWRITE,1040)NN,NE                                           STR02750
1040  FORMAT(' ',4X,'NO. OF NODES = ',I3,//,5X,                         STR02760
     1'NO. OF ELEMENTS = ',I2)                                          STR02770
      WRITE(NWRITE,1045)                                                STR02780
1045  FORMAT(' ',20X,'SUMMARY OF NODAL COORDINATES')                    STR02790
      WRITE(NWRITE,1050)                                                STR02800
1050  FORMAT(' ','NODE',5X,'Z(I)',11X,'R(I)',11X,'B(I)',                STR02810
     1 7X,'ABS FLOW ANG',3X,'REL FLOW ANG')                             STR02820
      WRITE(NWRITE,1060)(I,ZC(I),RC(I),B(I),ALP(I),BE(I),I=1,NN)        STR02830
1060  FORMAT(' ',I3,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6)       STR02840
      WRITE(NWRITE,1062)                                                STR02850
1062  FORMAT(' ',20X,'SYSTEM TOPOLOGY',//,2X,'ELEMENT',                 STR02860
     110X,'NODES',40X,'TYPE OF ELEMENT')                                STR02870
      WRITE(NWRITE,1063)(I,NODE(I,1),NODE(I,2),NODE(I,3),NODE(I,4)      STR02880
```

```
     1,NODE(I,5),NODE(I,6),NODE(I,7),NODE(I,8),NTE(I),I=1,NE)          STR02890
1063 FORMAT(' ',3X,I3,6X,I3,3X,I3,3X,I3,3X,I3,3X,I3,3X,I3,3X,I3,3X,I3, STR02900
     15X,I3)                                                            STR02910
     WRITE(NWRITE,1064)WDOT,RHOT,PT,TT, SPEED,UINLET,UOULET,RG,G,CP     STR02920
1064 FORMAT(' ',14X,'INLET THERMODYNAMIC VARIABLES ARE AS FOLLOWS',//, STR02930
     1 4X,'FLOW RATE = ',E13.6,' LBM/SEC',//,4X,'TOT DENSITY = ',E13.6,'STR02940
     2 LBM CU FT',//,4X,'TOT PRESSURE = ',E13.6,' PSI',//              STR02950
     3,4X,'TOT TEMPERATURE = ',E13.6,' DEG RANKINE',//,                STR02960
     4 4X,'ROTATIONAL SPEED = ',E13.6,' RPM',//,                       STR02970
     5 4X,'INLET U VELOCITY = ',E13.6,' FT/SEC',//,                    STR02980
     6 4X,'OUTLET U VELOCITY = ',E13.6,' FT/SEC',//,                   STR02990
     7 4X,'GAS CONSTANT = ',E13.6,//,                                  STR03000
     8 4X,'RATIO OF SPECIFIC HEATS = ',E13.6,//,                       STR03010
     9 4X,'SPECIFIC HEAT AT CONSTANT PRESSURE = ',E13.6)               STR03020
     WRITE(NWRITE,1065)RHOSTA,PSII                                      STR03030
1065 FORMAT(' ',4X,'STATIC DENSITY AT INLET = ',E13.6,//,4X,           STR03040
     1'INITIAL ESTIMATE OF PSI DISTRIBUTION = ',E13.6)                  STR03050
     WRITE(NWRITE,1070)                                                 STR03060
1070 FORMAT(' ',4X,'NODES WHERE PSI IS SPECIFIED',///,4X,               STR03070
     1'NODE NO.',10X,'PSI(I)')                                          STR03080
     WRITE(NWRITE,1080)(NPSIS(I),PSI(NPSIS(I)),I=1,NNSPSI)              STR03090
1080 FORMAT(' ',5X,I3,10X,E13.6)                                        STR03100
     WRITE(NWRITE,1090)                                                 STR03110
1090 FORMAT(' ',4X,'NODES WHERE F(R,Z) IS SPECIFIED',///,               STR03120
     14X,'NODE NO.')                                                    STR03130
     WRITE(NWRITE,1100)(NFS(I),I=1,NNFSP)                               STR03140
1100 FORMAT(' ',5X,I3,5X,I3,5X,I3,5X,I3,5X,I3,5X,I3,5X,I3,5X,I3,5X,I3) STR03150
     CALL MPLOT(RC,ZC,NODE,NN,NE)
C                                                                       STR03160
C               END OF INPUT ROUTINE.                                   STR03170
C                                                                       STR03180
C                                                                       STR03190
C                                                                       STR03200
C               BEGIN FIRST ITERATION AND INITIALIZE STREAM FUNCTION    STR03210
C               ITERATION COUNTER..................................KK.. STR03220
C                                                                       STR03230
1101 KK = 0                                                             STR03240
C                                                                       STR03250
C               CONVERT UINLET TO INCHES/SEC FOR COMPUTATIONAL COMPATIBISTR03260
C               LITY.                                                   STR03270
C                                                                       STR03280
     UINLET = UINLET*12.D0                                              STR03290
C                                                                       STR03300
C               NOW UINLET IS IN (INCHES/SEC)                           STR03310
C                                                                       STR03320
C                                                                       STR03330
C               FROM GIVEN DISTRIBUTION OF PSI AND INLET CONDITIONS,    STR03340
C               DETERMINE GEOMETRY OF STREAMLINES AND CALCULATE RADIAL  STR03350
```

```
C                 DISTRIBUTIONS OF WHIRL,ENTHALPY,ROTHALPY AND ENTROPY.
C
1105  CALL SLINE(UINLET,RC,PSI,WRL,H,UVEL,VVEL,TVEL,NODE,INLET
     1,NNODEI,NE,CP,TT,KK,NTE,ALP,WG,TWEL,BE,HS)
C
C                 CALCJLATE THE RIGHT HAND SIDE VECTOR, F(R,Z), FROM
C                 GIVEN RADIAL DISTRIBUTIONS OF WHIRL, ENTHALPY,ROTHALPY,
C                 AND ENTROPY.
C
      CALL FCAL(F,W,H,ZA,EA,UVEL,RC,ZC,WRL,TVEL,NFS,NODE,NN,NE,NNFSP
     1,TWEL,NTE)
C
C
C
C
C                 CYCLE FOR EACH ELEMENT AND CALCULATE ELEMENTAL MATRIX
1103  DO 400 II = 1,NE
      RC$(1) = RC(NODE(II,1))
      RC$(2) = RC(NODE(II,2))
      RC$(3) = RC(NJDE(II,3))
      RC$(4) = RC(NODE(II,4))
      RC$(5) = RC(NODE(II,5))
      RC$(6) = RC(NODE(II,6))
      RC$(7) = RC(NODE(II,7))
      RC$(8) = RC(NODE(II,8))
      ZC$(1) = ZC(NODE(II,1))
      ZC$(2) = ZC(NODE(II,2))
      ZC$(3) = ZC(NODE(II,3))
      ZC$(4) = ZC(NODE(II,4))
      ZC$(5) = ZC(NODE(II,5))
      ZC$(6) = ZC(NODE(II,6))
      ZC$(7) = ZC(NODE(II,7))
      ZC$(8) = ZC(NODE(II,8))
C
C
C                 CALCULATE BBAR
C
      BBAR = (B(NODE(II,1)) + B(NODE(II,2)) + B(NODE(II,3))
     1 + B(NJDE(II,4)) + B(NODE(II,5)) + B(NODE(II,6))
     2 + B(NODE(II,7)) + B(NODE(II,8)))/8.DO
      DO 300 K = 1,8
      DO 310 J = 1,8
      DO 320 I = 1,9
C
C                 CALCULATE APPROPIATE SHAPE FUNCTIONS
C
      CALL SHAPE(EA(I),ZA(I),SF)
C
```

```
C                CALCULATE JACOBIAN
C
      CALL JACOB(EA(I),ZA(I),D,E,RC$,ZC$,RJAC)
C
C                FIND RHOE AND R FOR NUMERICAL INTEGRATION.
C
      DO 330 L = 1,8
      RHOE(II) = RHOE(II) + SF(L)*RHO(NODE(II,L))
      R(II) = R(II) + SF(L)*RC(NODE(II,L))
330   CONTINUE
C
C                FIND DETERMINANT OF THE JACOBIAN.
C
      DETJ = RJAC(1,1)*RJAC(2,2) - RJAC(1,2)*RJAC(2,1)
      ROW(1,1) = D(K)
      ROW(1,2) = E(K)
      COL(1,1) = D(J)
      COL(2,1) = E(J)
C               FIND INVERSE OF JACOBIAN
      CALL DMINV(RJAC,2,A,L1,M1)
C
C               FIND TRANSPOSE OF INVERSE OF JACOBIAN
C
      DO 340 I1 = 1,2
      DO 340 J1 = 1,2
      TIJAC(J1,I1) = RJAC(I1,J1)
340   CONTINUE
C               CALCULATE ROW*TRANSPOSE(INV J)
C
      PROD1(1,1) = ROW(1,1)*TIJAC(1,1) + ROW(1,2)*TIJAC(2,1)
      PROD1(1,2) = ROW(1,1)*TIJAC(1,2) + ROW(1,2)*TIJAC(2,2)
C
C             CALCULATE PROD1 * INV(J)
C
      PROD2(1,1) = PROD1(1,1)*RJAC(1,1) + PROD1(1,2)*RJAC(2,1)
      PROD2(1,2) = PROD1(1,1)*RJAC(1,2) + PROD1(1,2) *RJAC(2,2)
C
C               CALCULATE PROD2 * COL
C
      PROD3(1) = PROD2(1,1)*COL(1,1) + PROD2(1,2)*COL(2,1)
C
C               MULTIPLY BY DET J
C
      FNZ1(1) = DETJ*PROD3(1)
C               CALCULATE K = 1/(RHO*R*B)
C
      FRHO = 1.D0/(RHOE(II)*R(II)*BBAR)
C               CALCULATE FINAL INTEGRAL EXPRESSION
```

```
C
      FNZ2(1) = ERHO*FNZ1(1)*W(I)
C
C              PERFORM GAUSSIAN QUADRATURF INTEGRATION
C
      EM$(K,J) = EM$(K,J) + FNZ2(1)*144.D0*12.D0
C
C              ZEROIZE OUT RHOE(II)
      RHOE(II) = 0.D0
      R(II) = 0.D0
320   CONTINUE
310   CONTINUE
300   CONTINUE
C
C     ASSEMBLE SYSTEM INFLUENCE MATRIX W/OUT REGARD FOR
C     BOUNDARY CONDITIONS
      N(1) = NODE(II,1)
      N(2) = NODE(II,2)
      N(3) = NODE(II,3)
      N(4) = NODE(II,4)
      N(5) = NODE(II,5)
      N(6) = NODE(II,6)
      N(7) = NODE(II,7)
      N(8) = NODE(II,8)
      DO 350 I$ = 1,8
      I = N(I$)
      DO 350 J$ = 1,8
      J = N(J$)
      EM(I,J) = EM(I,J) + EM$(I$,J$)
350   CONTINUE
C
      GO TO 361
2     WRITE(NWRITE,351)II
351   FORMAT(' ',///,15X,'ELEMENT MATRIX FOR ELEMENT ',I2)
      WRITE(NWRITE,360)((EM$(I,J),J=1,8),I=1,8)
360   FORMAT(' ',2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,
     12X,E13.6,2X,E13.6,2X,E13.6)
C
C              ZEROIZE OUT EM$( ) FOR NEXT ELEMENT
361   DO 370 I2 = 1,8
      DO 370 J2 = 1,8
      EM$(I2,J2) = 0.D0
370   CONTINUE
C
C              RECYCLE FOR NEXT ELEMENT
400   CONTINUE
C
C              MODIFY SYSTEM EQUATIONS TO INCLUDE BOUNDARY
```

```
C                PHENOMENA AND SAVE F(R,Z) FOR NEXT ITERATION IN NEEDED.  STR04800
C                                                                         STR04810
405     DO 410 I = 1,NN                                                   STR04820
        DO 410 J = 1,NTOTP                                                STR04830
        JX = NPSI(J)                                                      STR04840
        F(I) = F(I) - EM(I,JX)*PSI(JX)                                    STR04850
        EM(I,JX) = 0.D0                                                   STR04860
        EM(JX,I) = 0.D0                                                   STR04870
        EM(JX,JX) = 1.D0                                                  STR04880
        F(JX) = PSI(JX)                                                   STR04890
410     CONTINUE                                                          STR04900
        DO 415 I = 1,NN                                                   STR04910
        RHS(I) = F(I)                                                     STR04920
415     CONTINUE                                                          STR04930
C                                                                         STR04940
C                SOLVE SYSTEM OF EQUATIONS                                STR04950
C                                                                         STR04960
        CALL DSIMQ(EM,F,NN,KS)                                            STR04970
        GOTO1310                                                          STR04980
C                                                                         STR04990
C                                                                         STR05000
C                REPLACE PSI(I) WITH SOLUTION VECTOR AND                  STR05010
C                RESET F(I) WITH RHS(I).                                  STR05020
1131    DO 420 I = 1,NN                                                   STR05030
        PSI(I) = F(I)                                                     STR05040
        F(I) = RHS(I)                                                     STR05050
420     CONTINUE                                                          STR05060
C                                                                         STR05070
C                 IF KK GE 1, PERFORM UNDER RELAXATION BEFORE             STR05080
C                 COMPUTING NEW VELOCITY AND DENSITY DISTRIBUTION.        STR05090
C                                                                         STR05100
        IF(KK.EQ.0)GOTO423                                                STR05110
        DO 471 I = 1,NN                                                   STR05120
        IF(PSI(I).GT.PSI(1))GOTO472                                       STR05130
471     CONTINUE                                                          STR05140
        DO 473 I = 43,93                                                  STR05150
        PSI(I) = PSIO(I) + .2D0*(PSI(I) - PSIO(I))                        STR05160
473     CONTINUE                                                          STR05170
        GOTO423                                                           STR05180
472     DO 480 I = 43,93                                                  STR05190
        PSI(I) = PSIO(I) - .2D0*(PSI(I) - PSIO(I))                        STR05200
480     CONTINUE                                                          STR05210
C                                                                         STR05220
C                 FROM THE CURRENT DISTRIBUTION OF PSI AND DENSITY, CALCU-STR05230
C                 LATE THE 'V' AND 'U' VELOCITIES.  IN ADDITION, CALCULATESTR05240
C                 A NEW NODAL DENSITY DISTRIBUTION.                       STR05250
C                                                                         STR05260
423     CALL VEL(NE,NN,RC,NODE,G,RG,TT,RHOT,RHON,ZC,PSI,RHO,B,UINLET,     STR05270
       1UVEL,VVEL,RHOSTA,ALP,BE,H,WG)
```

```
      DO 4321  I  = 1,NN
      UVEL(I)  =  UVEL(I)/12.DO
      VVEL(I)  =  VVEL(I)/12.DO
4321  CONTINUE
      DO 2345  I  = 1,NN
      UVEL(I)  =  UVEL(I)*12.DO
      VVEL(I)  =  VVEL(I)*12.DO
2345  CONTINUE
C                COMPARE 'NEW STREAM FUNCTION' DISTRIBUTION WITH 'OLD
C                STREAM FUNCTION DISTRIBUTION.
C
500   X = 0.DO
      DO 430 I  = 1,NN
      IF(PSI(I).EQ.0.DO)GOTO421
      EPS = DABS((PSIO(I) - PSI(I))/PSI(I))
      GOTO422
421   EPS = DABS(PSIO(I) - PSI(I))
422   IF (X.GT.EPS)GOTO430
      X = EPS
430   CONTINUE
C
C
C          ASK IF STREAM FUNCTION CONVERGENCE CRITERION BEEN SATISFIED?
C              IF IT HAS..........WRITE RESULTS
C              IF NOT.............ITERATE ONCE MORE.
C
      KK = KK + 1
      IF(X.LE.1.D-01 )GOTO450
      WRITE (NWRITE,1400)KK,X
1400  FORMAT(' ','LARGEST EPS FOR ITERATION ',I2,' IS ',D19.12)
      IF (KK.LT.2)GOTO1500
      WRITE(NWRITE,1600)KK,X
1600  FORMAT(' ','PROGRAM TERMINATED ON ITERATION NO.',I3,/,
     1' RESULTS WHICH FOLLOW ARE FOR CONVERGENCE EPSILON = ',D19.12)
      GOTO1104
1500  WRITE(NWRITE,1102)KK
1102  FORMAT(' ','ITERATION NO. ',I3,' COMPLETE',/,' STREAM FUNCTION CON
     1VERGENCE NOT YET SATISFIED.',/,' NEXT ITERATION IS IN PROGRESS')
C
C
C                NEXT ITERATION
C                ZEROIZE STIFFNESS MATRIX AND RIGHT HAND SIDE VECTOR TO
C                PREPARE FOR NEXT ITERATION.
C
C
550   DO 460 I = 1,NN
      F(I) = 0.DO
      DO 460 J = 1,NN
```

```
      EM(I,J) = 0.D0
460   CONTINUE
C
C
C               REPLACE PSIO() WITH CURRENT VALUE OF PSI.  RETURN
C               TO 'CALL SLINE' STATEMENT AND RECLACULATE BOTH THE
C               STIFFNESS MATRIX AND RIGHT HANDRIGHT HAND
C               SIDE VECTOR AND PERFORM NEW CALCULATION FOR PSI.
C
      DO 470 I = 1,NN
      PSIO(I) = PSI(I)
470   CONTINUE
C
C               NEXT ITERATION
C
      GOTO1105
450   WRITE(NWRITE,1200)KK
1200  FORMAT(' ','STREAM FUNCTION CONVERGENCE CRITERION SATISFIED ON ITE
     1RATION  NUMBER ',I3,//,' RESULTS ARF AS FOLLOWS..................
     2.....')
C             WRITE OUT RESULTS
C     GOTO1111
C
C               CHANGE UNITS OF VELOCITY TO FT/SEC
C
1104  DO 600 I = 1,NN
      UVEL(I) = UVEL(I)/12.D0
      VVEL(I) = VVEL(I) /12.D0
      TVEL(I) = TVEL(I)/12.D0
      TWEL(I) = TWEL(I)/12.D0
600   CONTINUE
      WRITE(NWRITE,1110)
1110  FORMAT(' ',/////,20X,'FINITE ELEMENT RESULTS',///,
     1' NODE',5X,'PSI(I)',10X,'VZ',13X,'VR',12X,'R(I)',10X,'DENSITY')
      WRITE(NWRITE,1120)(I,PSI(I),UVEL(I),VVEL(I),RC(I),RHO(I),I=1,NN)
1120  FORMAT(' ',I3,2X,D13.6,2X,D13.6,2X,D13.6,2X,D13.6,2X,D13.6)
1111  WRITE(NWRITE,1121)
1121  FORMAT(' ','NODE',5X,'WRL(I)',10X,'HT',13X,'VT',12X,'WT')
      WRITE(NWRITE,1300)(I,WRL(I),H(I),TVEL(I),TWEL(I),I=1,NN)
1300  FORMAT(' ',I3,2X,D13.6,2X,D13.6,2X,D13.6,2X,D13.6)
1310  STOP
      DEBUG SUBCHK
      END
      SUBROUTINE FCAL(F,W,H,ZA,EA,UVEL,RC,ZC,WRL,TVEL,NFS,NODE,NN,NE
     1,NNFSP,TWEL,NTF)
      IMPLICIT REAL*8(A-H,P-Z)
      DIMENSION RJAC(2,2),F(107),W(9),D(8),E(8),SF(8),ZA(9),EA(9)
      DIMENSION UVEL(107),RC$(8),ZC$(8),RC(107),ZC(107),WRL(107)
```

```
      DIMENSION TVEL(107)
      DIMENSION H(107),TWEL(107),F$(8),NFS(107),NODE(28,8),N(8),L1(2)
      DIMENSION M1(2),NTE(40)
C
C               THIS SUBROUTINF CALCULATES THE RIGHT-HAND SIDE VECTOR
C               F(R,Z) FROM KNOWN RADIAL DISTRIBUTIONS OF WHIRL,ENTHALPY
C               ,ROTHALPY, AND ENTROPY.
C
C               ZEROIZE OUT F$() .
C
      DO 50 I = 1,8
      F$(I) = 0.D0
50    CONTINUE
C
C               CYCLE FOR EACH ELEMENT.
C
      DO 100 II = 1,NE
      RC$(1) = RC(NODE(II,1))
      RC$(2) = RC(NODE(II,2))
      RC$(3) = RC(NODE(II,3))
      RC$(4) = RC(NODE(II,4))
      RC$(5) = RC(NODE(II,5))
      RC$(6) = RC(NODE(II,6))
      RC$(7) = RC(NODE(II,7))
      RC$(8) = RC(NODE(II,8))
      ZC$(1) = ZC(NODE(II,1))
      ZC$(2) = ZC(NODE(II,2))
      ZC$(3) = ZC(NODE(II,3))
      ZC$(4) = ZC(NODE(II,4))
      ZC$(5) = ZC(NODE(II,5))
      ZC$(6) = ZC(NODE(II,6))
      ZC$(7) = ZC(NODE(II,7))
      ZC$(8) = ZC(NODE(II,8))
C
C               CYCLE FOR EACH LOCAL NODE.
C
      DO 200 I = 1,8
C
C               CHECK TO SEE IF PSI HAS BEEN SPECIFIED AT THE NODE....
C               IF SO ....  DO NOT CALCULATE A VALUE OF F(R,Z) FOR THE
C               NODE TO AVOID OVER SPECIFICATION AND CYCLE FOR THE NEXT
C               NODE.
C
      DO 210 L = 1,NNFSP
      LTEST = NODE(II,I) - NFS(L)
      IF(LTEST.EQ.0)GOTO220
210   CONTINUE
      GOTO200
```

```
C
C                CYCLE FOR EACH GAUSSIAN QUADRATURE INTEGRATION POINT.
C
220   DO 300 J = 1,9
C
C                FIND THE APPROPIATE SHAPE FUNCTIONS.
C
      CALL SHAPE(EA(J),ZA(J),SF)
C
C                FIND THE JACOBIAN.
C
      CALL JACOB(EA(J),ZA(J),D,E,RC$,ZC$,RJAC)
C
C                FIND THE DETERMINANT OF THE JACOBIAN.
C
      DETJ = RJAC(1,1)*RJAC(2,2) - RJAC(1,2)*RJAC(2,1)
C
C                FIND THE INVERSE OF THE JACOBIAN.
C
      CALL DMINV(RJAC,2,A,L1,M1)
C
C                FIND NI*UI,NI*V THETAI,NI*RI......
C
      SUMU = 0.D0
      SUMV = 0.D0
      SUMR = 0.D0
      DO 110 KK = 1,8
      SUMU = SUMU + SF(KK)*UVEL(NODE(II,KK))
      IF(NTE(II).EQ.2)GOTO105
      SUMV = SUMV + SF(KK)*TVEL(NODE(II,KK))
      GOTO106
105   SUMV = SUMV + SF(KK)*TWEL(NODE(II,KK))
106   SUMR = SUMR + SF(KK)*RC(NODE(II,KK))
110   CONTINUE
C
C                FIND D(WHIRL)/DR
C
      SUMW = 0.D0
      DO 120 KK = 1,8
      SUMW = SUMW + (RJAC(2,1)*D(KK) + RJAC(2,2)*E(KK))*WRL(NODE(II,KK))
120   CONTINUE
C
C                FIND D(ENTHALPY)/(DR)
C
      SUMH = 0.D0
121   DO 130 KK = 1,8
      SUMH = SUMH - (RJAC(2,1)*D(KK) + RJAC(2,2)*E(KK))*H(NODE(II,KK))*
     1 32.174D0*778.D0*144.D0
```

```
130    CONTINUE                                                          STR07200
C                                                                        STR07210
C              FIND F$(NODE(II,I))                                       STR07220
C                                                                        STR07230
131    XX = SUMH + (SUMV/SUMR)*SUMW                                      STR07240
       XX = (SF(I)/SUMU)*XX*DETJ                                         STR07250
       F$(I) =  F$(I) + XX*W(J)                                          STR07260
300    CONTINUE                                                          STR07270
200    CONTINUE                                                          STR07280
C                                                                        STR07290
C              ASSEMBLE RIGHT HAND SIDE VECTOR.                          STR07300
C                                                                        STR07310
       N(1) = NODE(II,1)                                                 STR07320
       N(2) = NODE(II,2)                                                 STR07330
       N(3) = NODE(II,3)                                                 STR07340
       N(4) = NODE(II,4)                                                 STR07350
       N(5) = NODE(II,5)                                                 STR07360
       N(6) = NODE(II,6)                                                 STR07370
       N(7) = NODE(II,7)                                                 STR07380
       N(8) = NODE(II,8)                                                 STR07390
       DO 400 I$ = 1,8                                                   STR07400
       I = N(I$)                                                         STR07410
       F(I) = F(I) + F$(I$)                                              STR07420
400    CONTINUE                                                          STR07430
C                                                                        STR07440
C              ZEROIZE OUT F$() FOR NEXT ELEMENTAL CALCULATION.          STR07450
C                                                                        STR07460
       DO 500 I = 1,8                                                    STR07470
       F$(I) = 0.D0                                                      STR07480
500    CONTINUE                                                          STR07490
C                                                                        STR07500
C              NEXT ELEMENT.                                             STR07510
C                                                                        STR07520
100    CONTINUE                                                          STR07530
       RETURN                                                            STR07540
       DEBUG SUBCHK                                                      STR07550
       END                                                               STR07560
       SUBROUTINE JACOB(E1,Z1,D,E,RC$,ZC$,RJAC)                          STR07570
       IMPLICIT REAL*8(A-H,P-Z)                                          STR07580
C                                                                        STR07590
C              THIS SUBROUTINE CALCULATES THE JACOBIAN MATRIX FOR SUBSESTR07600
C              QUENT NUMERICAL INTEGRATIONS BY GAUSSIAN QUADRATURE.      STR07610
       DIMENSION RJAC(2,2),D(8),E(8),RC$(8),ZC$(8)                       STR07620
       RJAC(1,1) = 0.D0                                                  STR07630
       RJAC(1,2) = 0.D0                                                  STR07640
       RJAC(2,1) = 0.D0                                                  STR07650
       RJAC(2,2) = 0.D0                                                  STR07660
       D(1) = (E1 + 2.D0*Z1 + 2.D0*Z1*E1 + E1*E1)/4.D0                   STR07670
```

```
      D(2) = -(Z1 + Z1*E1)                                              STR07680
      D(3) = (-E1 + 2.DO*Z1 + 2.DO*Z1*E1 - E1*E1)/4.DO                  STR07690
      D(4) = (-1.DO + E1*E1)/2.DO                                       STR07700
      D(5) = (E1 + 2.DO*Z1 - 2.DO*Z1*E1 - E1*E1)/4.DO                   STR07710
      D(6) = -Z1 + Z1*E1                                                STR07720
      D(7) = (-E1 + 2.DO*Z1 - 2.DO*Z1*E1 + E1*E1)/4.DO                  STR07730
      D(8) = (1.DO - E1*E1)/2.DO                                        STR07740
      E(1) = (Z1 + 2.DO*E1 + Z1*Z1 + 2.DO*Z1*E1)/4.DO                   STR07750
      E(2) = (1.DO - Z1*Z1)/2.DO                                        STR07760
      E(3) = (-Z1 + 2.DO*E1 + Z1*Z1 - 2.DO*Z1*E1)/4.DO                  STR07770
      E(4) = -E1 + E1*Z1                                                STR07780
      E(5) = (Z1 + 2.DO*E1 - Z1*Z1 - 2.DO*Z1*E1)/4.DO                   STR07790
      E(6) = (-1.DO + Z1*Z1)/2.DO                                       STR07800
      E(7) = (-Z1 + 2.DO*E1 - Z1*Z1 + 2.DO*Z1*E1)/4.DO                  STR07810
      E(8) = -(E1 + Z1*E1)                                              STR07820
      DO 100 I = 1,8                                                    STR07830
      RJAC(1,1) = RJAC(1,1) + D(I)*ZC$(I)                               STR07840
      RJAC(1,2) = RJAC(1,2) + D(I)*RC$(I)                               STR07850
      RJAC(2,1) = RJAC(2,1) + E(I)*ZC$(I)                               STR07860
      RJAC(2,2) = RJAC(2,2) + E(I)*RC$(I)                               STR07870
  100 CONTINUE                                                          STR07880
      RETURN                                                            STR07890
      DEBUG SUBCHK                                                      STR07900
      END                                                               STR07910
      SUBROUTINE VEL(NE,NN,RC,NODE,G,RG,TT,RHOT,RHON,ZC,PSI,RHO,B,UINLETSTR07920
     1,UVEL,VVEL,RHOSTA,ALP,BE,H,WG)                                    STR07930
C                                                                       STR07940
C              THIS SUBROUTINE CALCULATES U AND V VELOCITIES AND A NEW  STR07950
C              NODAL DENSITY FROM A KNOWN PSI DISTRIBUTION AT EACH OF   STR07960
C              THE NODES IN THE SYSTEM. IN ADDITION, THE CALL STATEMENTSTR07970
C              TRANSFERS THE NUMBER OF ELEMENTS,NODAL COORDINATES,LOCALSTR07980
C              NODE NUMBERS,RATIO OF SPECIFIC HEATS,GAS CONSTANT,TOTAL  STR07990
C              TEMPERATURE,TOTAL DENSITY,THE LATEST CALCULATED NODAL STSTR08000
C              STREAM FUNCTION,DENSITY DISTRIBUTION,NODAL BLOCKAGE FAC-STR08010
C              TOR,AND INLET VELOCITY CONDITIONS.                       STR08020
      IMPLICIT REAL*8(A-H,P-Z)                                          STR08030
      DIMENSION RJAC(2,2),PSI(107),D(8),E(8),UVEL(107),VVEL(107)        STR08040
      DIMENSION RC$(8),ZC$(8),RC(107),ZC(107),RHO(107),E1(8),Z1(8)      STR08050
      DIMENSION DR(8),DZ(8),B(107),RHON(107),H(107),ALP(107),BE(107)    STR08060
      DIMENSION NODE(28,8),L1(2)                                        STR08070
      DIMENSION M1(2)                                                   STR08080
      DATA Z1/1.DO,0.DO,-1.DO,-1.DO,-1.DO,0.DO,1.DO,1.DO/               STR08090
      DATA E1/1.DO,1.DO,1.DO,0.DO,-1.DO,-1.DO,-1.DO,0.DO/               STR08100
      DO 100 II = 1,NE                                                  STR08110
      RC$(1) = RC(NODE(II,1))                                           STR08120
      RC$(2) = RC(NODE(II,2))                                           STR08130
      RC$(3) = RC(NODE(II,3))                                           STR08140
      RC$(4) = RC(NODE(II,4))                                           STR08150
```

```
      RC$(5)  =  RC(NODE(II,5))
      RC$(6)  =  RC(NODE(II,6))
      RC$(7)  =  RC(NODE(II,7))
      RC$(8)  =  RC(NODE(II,8))
      ZC$(1)  =  ZC(NODE(II,1))
      ZC$(2)  =  ZC(NODE(II,2))
      ZC$(3)  =  ZC(NODE(II,3))
      ZC$(4)  =  ZC(NODE(II,4))
      ZC$(5)  =  ZC(NODE(II,5))
      ZC$(6)  =  ZC(NODE(II,6))
      ZC$(7)  =  ZC(NODE(II,7))
      ZC$(8)  =  ZC(NODE(II,8))
C
C
      DO 200 I = 1,8
C
C             CALCULATE JACOBIAN.
      CALL JACOB(F1(I),Z1(I),D,E,RC$,ZC$,RJAC)
C
C             FIND INVERSE OF JACOBIAN.
C
      CALL DMINV(RJAC,2,A,L1,M1)
C
C             FIND DN/DR AND DN/DZ
C
      DO 400 L = 1,8
      DZ(L) = RJAC(1,1)*D(L) + RJAC(1,2)*E(L)
      DR(L) = RJAC(2,1)*D(L) + RJAC(2,2)*E(L)
400   CONTINUE
C
C             CHECK TO SEE IF SOLUTION IS AT INLET
      IF(ZC$(I).EQ.0.D0)GOTO10
C
C             FIND D(PSI)/DR AND D(PSI)/DZ
      PSIR = DR(1)*PSI(NODE(II,1)) + DR(2)*PSI(NODE(II,2)) +
     1 DR(3)*PSI(NODE(II,3)) + DR(4)*PSI(NODE(II,4)) +
     2 DR(5)*PSI(NODE(II,5)) + DR(6)*PSI(NODE(II,6)) +
     3 DR(7)*PSI(NODE(II,7)) + DR(8)*PSI(NODE(II,8))
      PSIZ = DZ(1)*PSI(NODE(II,1))+DZ(2)*PSI(NODE(II,2))
     1 + DZ(3)*PSI(NODE(II,3)) + DZ(4)*PSI(NODE(II,4))
     2 + DZ(5)*PSI(NODE(II,5)) + DZ(6)*PSI(NODE(II,6))
     3 + DZ(7)*PSI(NODE(II,7)) + DZ(8)*PSI(NODE(II,8))
C             CALCULATE NEW NODAL DENSITY.
      ALPHA = ALP(NODE(II,I))
      BETA = BE(NODE(II,I))
      IDEN = 0
      XL = RHO(NODE(II,I)) - .01D0
      XR = RHO(NODE(II,I)) + .01D0
```

```
301   IDEN = IDEN + 1                                                    STR08640
      IF(NODE(II,I).LT.43)GOTO300                                        STR08650
      IF(NODE(II,I).GT.65)GOTO300                                        STR08660
      XM = (XL + XR)/2.D0                                                STR08670
      R1 = RC(NODE(II,I))                                                STR08680
      B1 = B(NODE(II,I))                                                 STR08690
      W1 = (PSIZ**2 + PSIR**2)/((XM*R1*B1)**2)                           STR08700
      W1 = (W1*144.D0*144.D0/32.174D0)/(DCOS(BETA)**2)                   STR08710
      U1 = (WG*R1)**2/(144.D0*32.174D0)                                  STR08720
      A1 = (G - 1.D0)*(W1 - U1)/(2.D0*G*RG*TT)                           STR08730
      A2 = DSQRT((PSIR**2+PSIZ**2)/((XM*R1*B1)**2))*DTAN(ALPHA)          STR08740
      A2 = (G - 1.D0)*(WG*R1)*A2/(G*RG*TT)                               STR08750
      A2 = A2*12.D0/32.174D0                                             STR08760
      RHON(NODE(II,I)) = RHOT*((1.D0-A2-A1)**(1.D0/(G-1.D0)))            STR08770
      GOTO315                                                            STR08780
300   A1 = (G - 1.D0)/(2.D0*G*RG*TT)                                     STR08790
      XM = (XL + XR)/2.D0                                                STR08800
      A1 = A1/((XM            *RC(NODE(II,I))*B(NODE(II,I)))**2)         STR08810
      A1 = A1*144.D0*144.D0/32.174D0                                     STR08820
      A1 = 1.D0 - A1*(PSIZ**2 + PSIR**2*(1.D0 + DTAN(ALPHA)**2))         STR08830
      A1 = A1**(1.D0/(G - 1.D0))                                         STR08840
      RHON(NODE(II,I)) = RHOT*A1                                         STR08850
315   EPS = RHON(NODE(II,I)) - XM                                        STR08860
      X4 = DABS(EPS)                                                     STR08870
      IF(X4.LT.1.D-06)GOTO250                                            STR08880
      IF(IDEN.EQ.10)GOTO250                                              STR08890
      IF(EPS.LT.0.D0)GOTO310                                             STR08900
      XL = XM                                                            STR08910
      GOTO301                                                            STR08920
310   XR = XM                                                            STR08930
      GOTO301                                                            STR08940
250   IF(XM.GT.6.D-02)GOTO255                                            STR08950
      XM = 0.6D-01                                                       STR08960
255   RHON(NODE(II,I)) = XM                                              STR08970
C                                                                        STR08980
C              FIND UVEL AND VVEL                                        STR08990
      UVEL(NODE(II,I)) = PSIR/(XM              *RC$(I)*B(NODE(II,I)))    STR09000
      UVEL(NODE(II,I)) = UVEL(NODE(II,I))*144.D0*12.D0                   STR09010
      VVEL(NODE(II,I)) = -PSIZ/(XM              *RC$(I)*B(NODE(II,I)))   STR09020
      VVEL(NODE(II,I)) = VVEL(NODE(II,I))*144.D0*12.D0                   STR09030
      GO TO 200                                                          STR09040
10    UVEL(NODE(II,I)) = UINLET                                          STR09050
      VVEL(NODE(II,I)) = 0.D0                                            STR09060
      RHON(NODE(II,I)) = RHOSTA                                          STR09070
200   CONTINUE                                                           STR09080
C                                                                        STR09090
C              NEXT ELEMENT                                              STR09100
100   CONTINUE                                                           STR09110
```

```
      DO 500 I = 1,NN                                                   STR09120
      RHO(I) = RHON(I)                                                  STR09130
500   CONTINUE                                                          STR09140
C                                                                       STR09150
C                                                                       STR09160
      RETURN                                                            STR09170
      DEBUG SUBCHK                                                      STR09180
      END                                                               STR09190
      SUBROUTINE SLINE(UINLET,RC,PSI,WRL,H,UVEL,VVEL,TVEL,NODE          STR09200
     1,INLET,NNODEI,NE,CP,TT,KK,NTE,ALP,WG,TWEL,BE,HS)                  STR09210
      IMPLICIT REAL*8(A-H,P-Z)                                          STR09220
      DIMENSION PSI(107),SF(8),UVEL(107),VVEL(107),RC(107),WRL(107)     STR09230
      DIMENSION TVEL(107)                                               STR09240
      DIMENSION H(107),ALP(107),TWEL(107),BE(107),HS(107)               STR09250
      DIMENSION E1(8),NODE(28,8)                                        STR09260
      DIMENSION INLET(10),NTE(40)                                       STR09270
      DATA E1/1.D0,1.D0,1.D0,0.D0,-1.D0,-1.D0,-1.D0,0.D0/               STR09280
C                                                                       STR09290
C              THIS SUBROUTINE CALCULATES THE THERMODYNAMIC VARIABLES   STR09300
C              THROUGHOUT THE SYSTEM FROM GIVEN INLET CONDITIONS AT THESTR09310
C              DUCT.  THIS IS DONE BY INITIALLY SEARCHING FOR THE       STR09320
C              STREAMLINES AT EACH NODE AND THEN USING THE ASSUMPTION   STR09330
C              THAT IN A DUCT REGION, THE SWHIRL (R*VTHETA) IS CONSTANTSTR09340
C              IN ROTOR,HREL IS CONSTANT; AND IN STATOR,H IS CONSTANT.  STR09350
C                                                                       STR09360
C                                                                       STR09370
C              FIND WHIRL AT INLET                                      STR09380
C              FIND V THETA AT INLET                                    STR09390
C              INSERT TOT ENTHALPY AT INLET NODES.                      STR09400
C              INSERT STATIC ENTHALPY AT INLET NODES                    STR09410
C                                                                       STR09420
      MM = 0                                                            STR09430
      THEVEL = UINLET*DTAN(ALP(1))                                      STR09440
      DO 100 I = 1,NNODEI                                               STR09450
      WRL(INLET(I)) = RC(INLET(I))*THEVEL                               STR09460
      H(INLET(I)) = CP*TT                                               STR09470
      HS(INLET(I)) = CP*TT-(UINLET**2)/7.20935D06                       STR09480
100   CONTINUE                                                          STR09490
C                                                                       STR09500
C              BEGIN WITH MID NODE OF FIRST ELEMENT AND THEN CYCLE      STR09510
C              THROUGH EACH ELEMENT.                                    STR09520
      DO 120 II = 1,NE                                                  STR09530
110   I = II                                                            STR09540
      IF(NTE( II).EQ.2)GOTO500                                          STR09550
      IF(NTE( II).EQ.3)GOTO600                                          STR09560
      GOTO111                                                           STR09570
C                                                                       STR09580
C              FIND HREL,WRL,AND TWEL AT LOC NODES 3,4,5(ROTOR).        STR09590
```

```
C
500   DO 501  J = 3,5
502   VZ  =  UVEL(NODE(II,J))
      VR  =  VVEL(NODE(II,J))
      A  =   BE(NODE(II,J))
      R  = RC(NODE(II,J))
      H(NODE( II,J)) =          (DSQRT(VZ*VZ+VR*VR)/DCOS(A))**2
      H(NODE( II,J))  =  H(NODE(II,J))  -  (WG*R)**2
      H(NODE( II,J))  =  H(NODE(II,J))/7.209035D06  + HS(NODE(II,J))
      TWEL(NODE(II,J))  =  DSQRT(VR*VR  +  VZ*VZ)*DTAN(A)
      WRL(NODE(II,J))  =  R*(WG*R  -  TWEL(NODE(II,J)))
501   CONTINUE
      GOTO111
C
C                FIND H,WRL,AND TVEL AT LOC NODES 3,4,5(STATOR).
C
600   DO 601  J = 3,5
      VZ  =  UVEL(NODE(II,J))
      VR  =  VVEL(NODE(II,J))
      A  = ALP(NODE(II,J))
      R  = RC(NODE(II,J))
      H(NODE( II,J))  =          (VR*VR+VZ*VZ)*(1.DO+DTAN(A)**2)
      H(NODE( II,J))  =  H(NODE(II,J))/7.209035D06  + HS(NODE(II,J))
      TVEL(NODE(II,J))  =  DSQRT(VR*VR+VZ*VZ)*DTAN(A)
      WRL(NODE(II,J))  =  R*TVEL(NODE(II,J))
601   CONTINUE
111   N  =  2
115   P  =  PSI(NODE(II,N))
      IT  =  0
      I  =  II
C                CHECK TO SEE IF P IS WITHIN THE PRESENT ELEMENT
130   IF(P.GE.PSI(NODE(I,5)))GOTO140
C                CHECK NEXT ELEMENT BELOW THE PRESENT ELEMENT.
      I  =  I  +  1
      GOTO130
140   IF(P.GT.PSI(NODE(I,4)))GOTO170
      EL  =  E1(4)
      ER  =  E1(5)
150   E  =  (EL  +  ER)/2.DO
      CALL  SHAPE(E,-1.DO,SF)
      PA  =  SF(3)*PSI(NODE(I,3))  +  SF(4)*PSI(NODE(I,4))
     1 + SF(5)*PSI(NODE(I,5))
C                CHECK FOR STREAMLINE CONVERGENCE
      EPS  =  DABS(PA  -  P)
      IF(EPS.LT.1.D-06)GOTO190
      IT  =  IT  +  1
      IF(IT.GT.10)GOTO190
      IF(PA.LT.P)GOTO160
```

```
      EL = E
      GOTO150
160   ER = E
      GOTO150
170   IF(P.GT.PSI(NODE(I,3)))GOTO180
165   EL = E1(3)
      ER = E1(4)
      GOTO150
C               CHECK NEXT ELEMENT ABOVE PRESENT ELEMENT
180   IF(KK.GT.0)GOTO185
      IF(I.EQ.1)GOTO165
      I = I -1
      GOTO140
185   IF(I.EQ.1.AND.N.EQ.2)GOTO187
      IF(I.EQ.1.AND.N.EQ.8)GOTO187
      I = I - 1
      GOTO140
187   I = II
      GOTO165
C               CONVERGENCE CRITERIA SATISFIED.
C
C               CHECK TO SEE WHAT TYPE OF ELEMENT.
C
190   IF(NTE(II).EQ.3)GOTO195
      IF(NTE(II).EQ.2)GOTO196
C               CALCULATE WHIRL AND STATIC ENTHALPY(DUCT)
      WRL(NODE(II,N)) = SF(3)*WRL(NODE(I,3)) + SF(4)*WRL(NODE(I,4))
     1 + SF(5)*WRL(NODE(I,5))
      HS(NODE(II,N)) = SF(3)*HS(NODE(I,3)) + SF(4)*HS(NODE(I,4))
     1 + SF(5)*HS(NODE(I,5))
      TVEL(NODE(II,N)) = WRL(NODE(II,N))/RC(NODE(II,N))
C
C               CALCULATE TOT ENTHALPY AT THE NODE.
C
      H(NODE(II,N)) = UVEL(NODE(II,N))**2*(1.D0 + DTAN(ALP(NODE(II,N)))*
     1*2)
      H(NODE(II,N)) = H(NODE(II,N)) + VVEL(NODE(II,N))**2
      H(NODE(II,N)) = HS(NODE(II,N)) + H(NODE(II,N))/7.20935D06
      GOTO199
C
C               CALCULATE TOT ENTHALPY AT NODE(STATOR).
C
195   H(NODE(II,N)) = SF(3)*H(NODE(I,3)) + SF(4)*H(NODE(I,4)) + SF(5)*
     1 H(NODE(I,5))
C
C               CALCULATE TVEL,SWHIRL,AND STATIC ENTHALPY AT NODE(STATOR
C
      VZ = UVEL(NODE(II,N))
```

```
      VR = VVEL(NODE(II,N))
      R = RC(NODE(II,N))
      TVEL(NODE(II,N)) = DSQRT(VR*VR+VZ*VZ)*DTAN(ALP(NODE(II,N)))
      WRL(NODE(II,N)) = R*TVEL(NODE(II,N))
      HS(NODE(II,N)) = H(NODE(II,N)) - (VR*VR+VZ*VZ)*(1.D0+DTAN(ALP(
     1 NODE(II,N)))**2)/7.209035D06
      GOTO199
C
C              CALCULATE HREL AT NODE(ROTOR).
C
196   H(NODE(II,N)) = SF(3)*H(NODE(I,3)) + SF(4)*H(NODE(I,4)) +
     1 SF(5)*H(NODE(I,5))
C
C              CALCULATE SWHIRL,TWEL,AND STATIC ENTHALPY AT NODE(ROTOR)
C
      VZ = UVEL(NODE(II,N))
      VR = VVEL(NODE(II,N))
      R = RC(NODE(II,N))
      TWEL(NODE(II,N)) = DSQRT(VR*VR+VZ*VZ)*DTAN( BE(NODE(II,N)))
      WRL(NODE(II,N)) = R*(WG*R - TWEL(NODE(II,N)))
      HS(NODE(II,N)) = H(NODE(II,N)) - (DSQRT(VR*VR+VZ*VZ)/DCOS(BE(NODE
     1 (II,N))))**2/7.209035D06   +(WG*R)**2/7.209035D06
      GOTO199
C
199   IF(N.EQ.1)GOTO120
      IF(N.GE.6)GOTO200
      N = 6
      GOTO115
200   N = N + 1
      IF(N.GT.8)GOTO210
      GOTO115
210   N = N- 8
      GOTO115
C              GO TO NEXT ELEMENT.
120   CONTINUE
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE SHAPE(E,Z,SF)
      IMPLICIT REAL*8(A-H,P-Z)
      DIMENSION SF(8)
      SF(1) = (Z*E + Z*Z + E*E + Z*Z*E + Z*E*E - 1.D0)/4.D0
      SF(2) = (1.D0 + E - Z*Z - Z*Z*E)/2.D0
      SF(3) = (-Z*E + Z*Z + E*E + Z*Z*E - Z*E*E - 1.D0)/4.D0
      SF(4) = (1.D0 - E*E - Z + Z*E*E)/2.D0
      SF(5) = (Z*E + Z*Z + E*E - Z*Z*E - Z*E*E - 1.D0)/4.D0
      SF(6) = (1.D0 - E - Z*Z + E*Z*Z)/2.D0
      SF(7) = (-Z*E + Z*Z + E*E - E*Z*Z + Z*E*E - 1.D0)/4.D0
```

```
      SF(8) = (1.D0 - E*E + Z -Z*E*E)/2.D0                          STR11040
      RETURN                                                        STR11050
      DEBUG SUBCHK                                                  STR11060
      END                                                           STR11070
       SUBROUTINE MPLOT(RC,ZC,NODE,NN,NE)                           MPL00010
       IMPLICIT REAL*8(A-H,P-Z)
       DIMENSION RC(NN),ZC(NN),OP1(9),OP2(9)
       DIMENSION O1(107),O2(107)
       DIMENSION NODE(NE,8)
       REAL*8 TITLE(3)/'GAVITO ','NASA    ','TASK 1 '/              MPL00040
       DATA OSCALE/.24/
       DO 100 L = 1,NN                                              MPL00060
       O1(L) = ZC(L)*OSCALE
       O2(L) = RC(L)*OSCALE
100    CONTINUE                                                     MPL00090
       CALL PLOTS                                                   MPL00100
       CALL PLOT(0.0,3.0,-3)                                        MPL00110
       DO 200 I = 1,NE                                              MPL00130
       NPE = 9                                                      MPL00140
       DO 300 J = 1,8                                               MPL00150
       OP1(J) = O1(NODE(I,J))
       OP2(J) = O2(NODE(I,J))
300    CONTINUE                                                     MPL00180
       CP1(NPE) = OP1(1)
       OP2(NPE) = OP2(1)
       CALL LINE(OP1,OP2,NPE,1,1)
200    CONTINUE                                                     MPL00220
      DO 400 K = 1,NN
       OL = FLOAT(K)
       CALL NUMBER(O1(K),O2(K),.07,OL,0,-1)
400    CONTINUE                                                     MPL00260
       CALL SYMBOL(0.0,5.0,.14,TITLE,0.0,24)                        MPL00270
       CALL PLOT(0.0,7.0,-3)                                        MPL00280
       CALL PLOTE                                                   MPL00290
       RETURN                                                       MPL00340
       CEBUG SUBCHK                                                 MPL00350
       END                                                          MPL00360
```

APPENDIX B

SAMPLE INPUT DATA

```
NASA TASK-1 TRANSONIC COMPRESSOR
    1C7          28
      1     0.0         18.878      1.
      2      .00        17.439      1.
      3      .00        16.0        1.
      4      .00        14.5        1.
      5      .00        13.0        1.
      6      .00        11.5        1.
      7      .00        10.0        1.
      8      .00         8.5495     1.
      9     0.           7.099      1.
     10     3.          18.484      1.
     11     3.          16.0        1.
     12     3.          13.0        1.0
     13     3.          10.0        1.
     14     3.           7.099      1.
     15     6.          18.409      1.
     16     6.          17.205      1.0
     17     6.0         16.0        1.
     18     6.0         14.5        1.
     19     6.0         13.0        1.
     20     6.0         11.5        1.
     21     6.0         10.0        1.
     22     6.0          8.5495     1.
     23     6.0          7.099      1.
     24     8.902       18.403      1.
     25     8.902       15.798      1.
     26     8.902       12.897      1.
     27     8.902        9.996      1.
     28     8.902        7.145      1.
     29    11.804       18.397      1.
     30    11.804       16.996      1.
     31    11.804       15.595      1.
     32    11.804       14.194      1.
     33    11.804       12.794      1.
     34    11.804       11.393      1.
     35    11.804        9.992      1.
     36    11.804        8.591      1.
     37    11.804        7.19       1.
     38    15.5         18.37       1.
     39    15.5         15.835      1.
     40    15.5         13.3        1.
     41    15.5         10.765      1.
     42    15.5          8.23       1.
     43    18.511       18.25     .9921628
     44    18.447       17.161    .991016
     45    18.383       16.072    .989867
     46    18.319       14.855    .988101
```

47	18.256	13.638	.986334
48	18.192	12.659	.984340
49	18.128	11.68	.982346
50	18.064	10.403	.978950
51	18.	9.125	.975553
52	19.522	18.01	.954283
53	19.524	16.037	.931421
54	19.527	13.706	.896215
55	19.528	11.778	.858371
56	19.53	9.634	.792201
57	20.533	17.871	.992424
58	20.599	16.937	.991146
59	20.665	16.002	.989867
60	20.731	14.888	.988100
61	20.797	13.773	.986336
62	20.862	12.824	.984242
63	20.928	11.875	.982148
64	20.994	11.009	.989852
65	21.06	10.142	.997555
66	21.692	17.838	1.
67	21.802	15.822	1.
68	21.911	14.	1.
69	22.031	12.286	1.
70	22.13	10.348	1.
71	22.85	17.836	.988014
72	22.904	16.711	.987362
73	22.958	15.586	.987650
74	22.962	14.859	.986970
75	23.028	14.132	.986091
76	23.062	13.415	.985934
77	23.096	12.698	.984777
78	23.148	11.625	.983765
79	23.2	10.553	.982754
80	24.625	17.836	.902611
81	24.615	15.642	.902895
82	24.610	14.228	.904725
83	24.605	12.852	.903892
84	24.6	10.831	.902994
85	26.4	17.836	.988014
86	26.336	16.767	.987362
87	26.273	15.697	.987650
88	26.232	15.011	.986970
89	26.191	14.325	.986091
90	26.152	13.665	.98543
91	26.113	13.005	.98477
92	26.056	12.057	.983762
93	26.0	11.109	.982754
94	29.7	17.839	1.

```
        95   29.65        15.949      1.
        96   29.6         14.413      1.
        97   29.55        12.900      1.
        98   29.5         11.14       1.
        99   33.          17.836      1.
       100   33.          17.018      1.
       101   33.          16.2        1.
       102   33.          15.35       1.
       103   33.          14.5        1.
       104   33.          13.65       1.
       105   33.          12.8        1.
       106   33.          11.985      1.
       107   33.          11.17       1.
   1    15   10     1     2     3    11    17    16
   2    17   11     3     4     5    12    19    18
   3    19   12     5     6     7    13    21    20
   4    21   13     7     8     9    14    23    22
   5    29   24    15    16    17    25    31    30
   6    31   25    17    18    19    26    33    32
   7    33   26    19    20    21    27    35    34
   8    35   27    21    22    23    28    37    36
   9    43   38    29    30    31    39    45    44
  10    45   39    31    32    33    40    47    46
  11    47   40    33    34    35    41    49    48
  12    49   41    35    36    37    42    51    50
  13    57   52    43    44    45    53    59    58
  14    59   53    45    46    47    54    61    60
  15    61   54    47    48    49    55    63    62
  16    63   55    49    50    51    56    65    64
  17    71   66    57    58    59    67    73    72
  18    73   67    59    60    61    68    75    74
  19    75   68    61    62    63    69    77    76
  20    77   69    63    64    65    70    79    78
  21    85   80    71    72    73    81    87    86
  22    87   81    73    74    75    82    89    88
  23    89   82    75    76    77    83    91    90
  24    91   83    77    78    79    84    93    92
  25    99   94    85    86    87    95   101   100
  26   101   95    87    88    89    96   103   102
  27   103   96    89    90    91    97   105   104
  28   105   97    91    92    93    98   107   106
         1          1
         2          1
         3          1
         4          1
         5          1
         6          1
         7          1
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8	1
9	1
10	1
11	1
12	1
13	2
14	2
15	2
16	2
17	1
18	1
19	1
20	1
21	3
22	3
23	3
24	3
25	1
26	1
27	1
28	1

43	.755553
44	.755379
45	.755029
46	.721694
47	.688183
48	.63146
49	.574737
50	.475777
51	.376817
52	.723788
53	.703717
54	.710175
55	.670003
56	.624479
57	.692198
58	.672301
59	.652404
60	.692372
61	.732166
62	.748921
63	.765501
64	.871094
65	.871966
66	.692198
67	.652404
68	.732166
69	.75721

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                    70    .871967
                    71    .692198
                    72    .672301
                    73    .652404
                    74    .692372
                    75    .732166
                    76    .740544
                    77    .748921
                    78    .810444
                    79    .871967
                    80    .346099
                    81    .326202
                    82    .36617
                    83    .40300
                    84    .435983
        STOP
                    43    1.08
                    44    1.0369
                    45    .993791
                    46    .950681
                    47    .907571
                    48    .881915
                    49    .856259
                    50    .823534
                    51    .790809
                    52   1.040565
                    53    .967785
                    54    .831475
                    55    .707731
                    56    .477173
                    57   1.001121
                    58    .97145
                    59    .941605
                    60    .848405
                    61    .755204
                    62    .657117
                    63    .559029
                    64    .361283
                    65    .163537
        STOP
107.6     201.482573318.846572  .08192     .08        15.0546    499.38
4359.5
53.35     1.4        .240
17.1250719
                    1017.1250719
                    1517.1250719
                    2417.1250719
                    2917.1250719
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3817.1250719
4317.1250719
5217.1250719
5717.1250719
6617.1250719
7117.1250719
8017.1250719
8517.1250719
9417.1250719
14  0.0
23  0.0
28  0.0
37  0.0
42  0.0
51  0.0
56   0.0
65  0.0
70  0.0
79   0.0
84   0.0
93   0.0
98  0.0
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STOP

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STOP

APPENDIX C

SAMPLE OUTPUT LISTING

NASA TASK-1 TRANSONIC COMPRESSOR

NO. OF NODES = 107

NO. OF ELEMENTS = 28

SUMMARY OF NODAL COORDINATES

NODE	Z(I)	R(I)	B(I)	ABS FLOW ANG	REL FLOW ANG
1	0.0	0.188780D 02	0.100000D 01	0.0	0.0
2	0.0	0.174390D 02	0.100000D 01	0.0	0.0
3	0.0	0.160000D 02	0.100000D 01	0.0	0.0
4	0.0	0.145000D 02	0.100000D 01	0.0	0.0
5	0.0	0.130000D 02	0.100000D 01	0.0	0.0
6	0.0	0.115000D 02	0.100000D 01	0.0	0.0
7	0.0	0.100000D 02	0.100000D 01	0.0	0.0
8	0.0	0.854950D 01	0.100000D 01	0.0	0.0
9	0.0	0.709900D 01	0.100000D 01	0.0	0.0
10	0.300000D 01	0.184840D 02	0.100000D 01	0.0	0.0
11	0.300000D 01	0.160000D 02	0.100000D 01	0.0	0.0
12	0.300000D 01	0.130000D 02	0.100000D 01	0.0	0.0
13	0.300000D 01	0.100000D 02	0.100000D 01	0.0	0.0
14	0.300000D 01	0.709900D 01	0.100000D 01	0.0	0.0
15	0.600000D 01	0.184090D 02	0.100000D 01	0.0	0.0
16	0.600000D 01	0.172050D 02	0.100000D 01	0.0	0.0
17	0.600000D 01	0.160000D 02	0.100000D 01	0.0	0.0
18	0.600000D 01	0.145000D 02	0.100000D 01	0.0	0.0
19	0.600000D 01	0.130000D 02	0.100000D 01	0.0	0.0
20	0.600000D 01	0.115000D 02	0.100000D 01	0.0	0.0
21	0.600000D 01	0.100000D 02	0.100000D 01	0.0	0.0
22	0.600000D 01	0.854950D 01	0.100000D 01	0.0	0.0
23	0.600000D 01	0.709900D 01	0.100000D 01	0.0	0.0
24	0.890200D 01	0.184030D 02	0.100000D 01	0.0	0.0
25	0.890200D 01	0.157980D 02	0.100000D 01	0.0	0.0
26	0.890200D 01	0.128970D 02	0.100000D 01	0.0	0.0
27	0.890200D 01	0.999600D 01	0.100000D 01	0.0	0.0
28	0.890200D 01	0.714500D 01	0.100000D 01	0.0	0.0
29	0.118040D 02	0.183970D 02	0.100000D 01	0.0	0.0
30	0.118040D 02	0.169960D 02	0.100000D 01	0.0	0.0
31	0.118040D 02	0.155950D 02	0.100000D 01	0.0	0.0
32	0.118040D 02	0.141940D 02	0.100000D 01	0.0	0.0
33	0.118040D 02	0.127940D 02	0.100000D 01	0.0	0.0
34	0.118040D 02	0.113930D 02	0.100000D 01	0.0	0.0
35	0.118040D 02	0.999200D 01	0.100000D 01	0.0	0.0
36	0.118040D 02	0.859100D 01	0.100000D 01	0.0	0.0
37	0.118040D 02	0.719000D 01	0.100000D 01	0.0	0.0
38	0.155000D 02	0.183700D 02	0.100000D 01	0.0	0.0
39	0.155000D 02	0.158350D 02	0.100000D 01	0.0	0.0
40	0.155000D 02	0.133000D 02	0.100000D 01	0.0	0.0
41	0.155000D 02	0.107650D 02	0.100000D 01	0.0	0.0
42	0.155000D 02	0.823000D 01	0.100000D 01	0.0	0.0

43	0.185110D 02	0.182500D 02	0.992163D 00	0.755553D 00	0.108000D 01
44	0.184470D 02	0.171610D 02	0.991016D 00	0.755379D 00	0.103690D 01
45	0.183830D 02	0.160720D 02	0.989867D 00	0.755029D 00	0.993791D 00
46	0.183190D 02	0.148550D 02	0.988101D 00	0.721694D 00	0.950681D 00
47	0.182560D 02	0.136380D 02	0.986334D 00	0.688183D 00	0.907571D 00
48	0.181920D 02	0.126590D 02	0.984340D 00	0.631460D 00	0.881915D 00
49	0.181280D 02	0.116800D 02	0.982346D 00	0.574737D 00	0.856259D 00
50	0.180640D 02	0.104030D 02	0.978950D 00	0.475777D 00	0.823534D 00
51	0.180000D 02	0.912500D 01	0.975553D 00	0.376817D 00	0.790809D 00
52	0.195220D 02	0.180100D 02	0.954283D 00	0.723788D 00	0.104057D 01
53	0.195240D 02	0.160370D 02	0.931421D 00	0.703717D 00	0.967785D 00
54	0.195270D 02	0.137060D 02	0.896215D 00	0.710175D 00	0.831475D 00
55	0.195280D 02	0.117780D 02	0.858371D 00	0.670003D 00	0.707731D 00
56	0.195300D 02	0.963400D 01	0.792201D 00	0.624479D 00	0.477173D 00
57	0.205330D 02	0.178710D 02	0.992424D 00	0.692198D 00	0.100112D 01
58	0.205990D 02	0.169370D 02	0.991145D 00	0.672301D 00	0.971450D 00
59	0.206650D 02	0.160020D 02	0.989867D 00	0.652404D 00	0.941605D 00
60	0.207310D 02	0.148880D 02	0.988100D 00	0.692372D 00	0.848405D 00
61	0.207970D 02	0.137730D 02	0.986336D 00	0.732166D 00	0.755204D 00
62	0.208620D 02	0.128240D 02	0.984242D 00	0.748921D 00	0.657117D 00
63	0.209280D 02	0.118750D 02	0.982148D 00	0.765501D 00	0.559029D 00
64	0.209940D 02	0.110090D 02	0.989852D 00	0.871094D 00	0.361283D 00
65	0.210600D 02	0.101420D 02	0.997555D 00	0.871966D 00	0.163537D 00
66	0.216920D 02	0.178380D 02	0.100000D 01	0.692198D 00	0.0
67	0.218020D 02	0.158220D 02	0.100000D 01	0.652404D 00	0.0
68	0.219110D 02	0.140000D 02	0.100000D 01	0.732166D 00	0.0
69	0.220310D 02	0.122860D 02	0.100000D 01	0.757210D 00	0.0
70	0.221300D 02	0.103480D 02	0.100000D 01	0.871967D 00	0.0
71	0.228500D 02	0.178360D 02	0.988014D 00	0.692198D 00	0.0
72	0.229040D 02	0.167110D 02	0.987362D 00	0.672301D 00	0.0
73	0.229580D 02	0.155860D 02	0.987650D 00	0.652404D 00	0.0
74	0.229620D 02	0.148590D 02	0.986970D 00	0.692372D 00	0.0
75	0.230280D 02	0.141320D 02	0.986091D 00	0.732166D 00	0.0
76	0.230620D 02	0.134150D 02	0.985934D 00	0.740544D 00	0.0
77	0.230960D 02	0.126980D 02	0.984777D 00	0.748921D 00	0.0
78	0.231480D 02	0.116250D 02	0.983765D 00	0.810444D 00	0.0
79	0.232000D 02	0.105530D 02	0.982754D 00	0.871967D 00	0.0
80	0.246250D 02	0.178360D 02	0.902611D 00	0.346099D 00	0.0
81	0.246150D 02	0.156420D 02	0.902895D 00	0.326202D 00	0.0
82	0.246100D 02	0.142280D 02	0.904725D 00	0.366170D 00	0.0
83	0.246050D 02	0.128520D 02	0.903892D 00	0.403000D 00	0.0
84	0.246000D 02	0.108310D 02	0.902994D 00	0.435983D 00	0.0
85	0.264000D 02	0.178360D 02	0.988014D 00	0.0	0.0
86	0.263360D 02	0.167670D 02	0.987362D 00	0.0	0.0
87	0.262730D 02	0.156970D 02	0.987650D 00	0.0	0.0
88	0.262320D 02	0.150110D 02	0.986970D 00	0.0	0.0
89	0.261910D 02	0.143250D 02	0.986091D 00	0.0	0.0
90	0.261520D 02	0.136650D 02	0.985430D 00	0.0	0.0

91	0.261130D 02	0.130050D 02	0.984770D 00	0.0	0.0
92	C.260560D 02	0.120570D 02	0.983762D 00	0.0	0.0
93	0.260000D 02	0.111090D 02	0.982754D 00	0.0	0.0
94	0.297000D 02	0.178390D 02	0.100000D 01	0.0	0.0
95	0.296500D 02	0.159490D 02	0.100000D 01	0.0	0.0
96	0.296000D 02	0.144130D 02	0.100000D 01	0.0	0.0
97	0.295500D 02	0.129000D 02	0.100000D 01	0.0	0.0
98	0.295000D 02	0.111400D 02	0.100000D 01	0.0	0.0
99	0.330000D 02	0.178360D 02	0.100000D 01	0.0	0.0
100	0.330000D 02	0.170180D 02	0.100000D 01	0.0	0.0
101	C.330000D 02	0.162000D 02	0.100000D 01	0.0	0.0
102	0.330000D 02	0.153500D 02	0.100000D 01	0.0	0.0
103	0.330000D 02	0.145000D 02	0.100000D 01	0.0	0.0
104	0.330000D 02	0.136500D 02	0.100000D 01	0.0	0.0
105	C.330000D 02	0.128000D 02	0.100000D 01	0.0	0.0
106	0.330000D 02	0.119850D 02	0.100000D 01	0.0	0.0
107	0.330000D 02	0.111700D 02	0.100000D 01	0.0	0.0

SYSTEM TOPOLOGY

ELEMENT	NODES								TYPE OF ELEMENT
1	15	10	1	2	3	11	17	16	1
2	17	11	3	4	5	12	19	18	1
3	19	12	5	6	7	13	21	20	1
4	21	13	7	8	9	14	23	22	1
5	29	24	15	16	17	25	31	30	1
6	31	25	17	18	19	26	33	32	1
7	33	26	19	20	21	27	35	34	1
8	35	27	21	22	23	28	37	36	1
9	43	38	29	30	31	39	45	44	1
10	45	39	31	32	33	40	47	46	1
11	47	40	33	34	35	41	49	48	1
12	49	41	35	36	37	42	51	50	1
13	57	52	43	44	45	53	59	58	2
14	59	53	45	46	47	54	61	60	2
15	61	54	47	48	49	55	63	62	2
16	63	55	49	50	51	56	65	64	2
17	71	66	57	58	59	67	73	72	1
18	73	67	59	60	61	68	75	74	1
19	75	68	61	62	63	69	77	76	1
20	77	69	63	64	65	70	79	78	1
21	85	80	71	72	73	81	87	86	3
22	87	81	73	74	75	82	89	88	3
23	89	82	75	76	77	83	91	90	3
24	91	83	77	78	79	84	93	92	3
25	99	94	85	86	87	95	101	100	1
26	101	95	87	88	89	96	103	102	1
27	103	96	89	90	91	97	105	104	1
28	105	97	91	92	93	98	107	106	1

INLET THERMODYNAMIC VARIABLES ARE AS FOLLOWS

FLOW RATE = 0.107600D 03 LBM/SEC

TOT DENSITY = 0.819200D-01 LBM CU FT

TOT PRESSURE = 0.150546 D 02 PSI

TOT TEMPERATURE = 0.499380D 03 DEG RANKINE

ROTATIONAL SPEED = 0.435950D 04 RPM

INLET U VELOCITY = 0.201483D 03 FT/SEC

OUTLET U VELOCITY = 0.318847D 03 FT/SEC

GAS CONSTANT = 0.533500D 02

RATIO OF SPECIFIC HEATS = 0.140000D 01

SPECIFIC HEAT AT CONSTANT PRESSURE = 0.240000D 00
STATIC DENSITY AT INLET = 0.800000D-01

INITIAL ESTIMATE OF PSI DISTRIBUTION = 0.171251D 02
NODES WHERE PSI IS SPECIFIED

NODE NO.	PSI(I)
10	0.171251D 02
15	0.171251D 02
24	0.171251D 02
29	0.171251D 02
38	0.171251D 02
43	0.171251D 02
52	0.171251D 02
57	0.171251D 02
66	0.171251D 02
71	0.171251D 02
80	0.171251D 02
85	0.171251D 02
94	0.171251D 02
14	0.0
23	0.0
28	0.0
37	0.0
42	0.0
51	0.0
56	0.0

```
   65            0.0
   70            0.0
   79            0.0
   84            0.0
   93            0.0
   98            0.0
    1            0.171251D 02
    2            0.142002D 02
    3            0.115071D 02
    4            0.894662D 01
    5            0.663797D 01
    6            0.458117D 01
    7            0.277622D 01
    8            0.127036D 01
    9            0.0
   99            0.171251D 02
  100            0.145999D 02
  101            0.121933D 02
  102            0.981814D 01
  103            0.757093D 01
  104            0.545171D 01
  105            0.346047D 01
  106            0.167140D 01
  107            0.0
 NODES WHERE F(R,Z) IS SPECIFIED

NODE NO.
   11     12     13     16     17     18     19     20     21
   22     25     26     27     30     31     32     33     34
   35     36     39     40     41     44     45     46     47
   48     49     50     53     54     55     58     59     60
   61     62     63     64     67     68     69     72     73
   74     75     76     77     78     81     82     83     86
   87     88     89     90     91     92     95     96     97
LARGEST EPS FOR ITERATION  1 IS  0.1360478975290D 02
ITERATION NO.   1 COMPLETE
STREAM FUNCTION CONVERGENCE NOT YET SATISFIED.
NEXT ITERATION IS IN PROGRESS
LARGEST EPS FOR ITERATION  2 IS  0.442913378700D 00
PROGRAM TERMINATED ON ITERATION NO.  2
RESULTS WHICH FOLLOW ARE FOR CONVERGENCE EPSILON =  0.442913378700D 00


              FINITE ELEMENT RESULTS
```

NODE	PSI(I)	VZ	VR	R(I)	DENSITY
1	0.171251D 02	0.201483D 03	0.0	0.188780D 02	0.800000D-01
2	0.142002D 02	0.201483D 03	0.0	0.174390D 02	0.800000D-01
3	0.115071D 02	0.201483D 03	0.0	0.160000D 02	0.800000D-01
4	0.894662D 01	0.201483D 03	0.0	0.145000D 02	0.800000D-01
5	0.663797D 01	0.201483D 03	0.0	0.130000D 02	0.800000D-01
6	0.458117D 01	0.201483D 03	0.0	0.115000D 02	0.800000D-01
7	0.277622D 01	0.201483D 03	0.0	0.100000D 02	0.800000D-01
8	0.127036D 01	0.201483D 03	0.0	0.854950D 01	0.800000D-01
9	0.0	0.201483D 03	0.0	0.709900D 01	0.800000D-01
10	0.171251D 02	0.220059D 03	-0.172013D 02	0.184840D 02	0.802734D-01
11	0.118445D 02	0.210036D 03	-0.114073D 02	0.160000D 02	0.804297D-01
12	0.678256D 01	0.206122D 03	-0.730155D 01	0.130000D 02	0.804687D-01
13	0.282503D 01	0.202621D 03	-0.272330D 01	0.100000D 02	0.805078D-01
14	0.0	0.205347D 03	0.0	0.709900D 01	0.804883D-01
15	C.171251D 02	0.211902D 03	-0.438116D 00	0.184090D 02	0.803906D-01
16	0.145630D 02	0.216573D 03	0.465115D 01	0.172050D 02	0.803125D-01
17	0.121188D 02	0.216545D 03	-0.158781D 01	0.160000D 02	0.803125D-01
18	C.937887D 01	0.212585D 03	-0.679883D 01	0.145000D 02	0.803906D-01
19	0.695622D 01	0.213867D 03	-0.684264D 01	0.130000D 02	0.803516D-01
20	0.477052D 01	0.212284D 03	-0.673067D 01	0.115000D 02	0.803906D-01
21	0.286757D 01	0.204643D 03	-0.564721D 01	0.100000D 02	0.805078D-01
22	0.132090D 01	0.206900D 03	0.225277D 01	0.854950D 01	0.804687D-01
23	0.0	0.210041D 03	0.336558D 01	0.709900D 01	0.804297D-01
24	0.171251D 02	0.227675D 03	-0.470728D 00	0.184030D 02	0.801562D-01
25	0.119735D 02	0.205688D 03	-0.166029D 02	0.157980D 02	0.804687D-01
26	0.702133D 01	0.220263D 03	-0.148156D 02	0.128970D 02	0.802539D-01
27	0.292664D 01	0.226396D 03	-0.223632D 01	0.999600D 01	0.801953D-01
28	0.0	0.198306D 03	0.310921D 01	0.714500D 01	0.805664D-01
29	0.171251D 02	0.226429D 03	0.109332D 01	0.183970D 02	0.801758D-01
30	0.142776D 02	0.183610D 03	-0.795651D 02	0.169960D 02	0.805469D-01
31	0.122341D 02	0.206530D 03	-0.166455D 03	0.155950D 02	0.795312D-01
32	0.974105D 01	0.228187D 03	-0.172725D 03	0.141940D 02	0.791406D-01
33	0.724815D 01	0.241888D 03	-0.179660D 03	0.127940D 02	0.788672D-01
34	0.498124D 01	0.245654D 03	-0.130826D 03	0.113930D 02	0.792969D-01
35	0.292974D 01	0.245873D 03	-0.662508D 02	0.999200D 01	0.797266D-01
36	0.124448D 01	0.218274D 03	-0.183315D 02	0.859100D 01	0.802930D-01
37	0.0	0.181316D 03	0.469758D 02	0.719000D 01	0.807422D-01
38	0.171251D 02	0.953714D 02	-0.209029D 01	0.183700D 02	0.816016D-01
39	0.156118D 02	0.211283D 03	0.488500D-01	0.158350D 02	0.803906D-01
40	0.107447D 02	0.341329D 03	0.124478D 02	0.133000D 02	0.780078D-01
41	0.467601D 01	0.359152D 03	0.419968D 02	0.107650D 02	0.775391D-01
42	0.0	0.365127D 03	0.114029D 03	0.823000D 01	0.770312D-01
43	0.171251D 02	0.188915D 03	-0.542827D 02	0.182500D 02	0.836719D-01
44	0.150238D 02	0.182983D 03	-0.363786D 02	0.171610D 02	0.840625D-01
45	0.131222D 02	0.181377D 03	-0.358019D 02	0.160720D 02	0.835156D-01

46	0.109989D 02	0.217508D 03	-0.332291D 02	0.148550D 02	0.804687D-01
47	0.874510D 01	0.263002D 03	-0.321793D 02	0.136380D 02	0.771484D-01
48	0.684972D 01	0.300928D 03	-0.128776D 02	0.126590D 02	0.751172D-01
49	C.490441D 01	0.357638D 03	0.801256D 01	0.116800D 02	0.718555D-01
50	0.237289D 01	0.385140D 03	0.629724D 02	0.104030D 02	0.710547D-01
51	0.0	0.433195D 03	0.144257D 03	0.912500D 01	0.681250D-01
52	0.171251D 02	0.242434D 03	-0.454414D 02	0.180100D 02	0.800000D-01
53	0.132234D 02	0.208945D 03	0.543975D 01	0.160370D 02	0.826172D-01
54	0.883947D 01	0.334585D 03	0.477602D 02	0.137060D 02	0.726172D-01
55	0.476760D 01	0.538251D 03	0.107145D 03	0.117780D 02	0.600391D-01
56	0.0	0.684311D 03	0.227433D 03	0.963400D 01	0.600391D-01
57	0.171251D 02	0.281327D 03	-0.117674D 02	0.178710D 02	0.800000D-01
58	0.147764D 02	0.236773D 03	0.134070D 02	0.169370D 02	0.817578D-01
59	0.128904D 02	0.217194D 03	0.529913D 02	0.160020D 02	0.825391D-01
6C	0.106224D 02	0.252037D 03	0.812320D 02	0.148880D 02	0.789453D-01
61	0.827315D 01	0.303197D 03	0.114025D 03	0.137730D 02	0.743359D-01
62	0.619073D 01	0.341721D 03	0.132514D 03	0.128240D 02	0.717969D-01
63	0.408225D 01	0.401864D 03	0.150547D 03	0.118750D 02	0.681836D-01
64	0.207387D 01	0.491840D 03	0.147459D 03	0.110090D 02	0.618750D-01
65	0.0	0.567354D 03	0.109494D 03	0.101420D 02	0.600391D-01
66	0.171251D 02	0.302506D 03	-0.456957D 01	0.178380D 02	0.767383D-01
67	0.120681D 02	0.255341D 03	0.349385D 02	0.158220D 02	0.783984D-01
68	0.798850D 01	0.300666D 03	0.802308D 02	0.140000D 02	0.762500D-01
69	0.417127D 01	0.334517D 03	0.980101D 02	0.122860D 02	0.745703D-01
70	0.0	0.445955D 03	0.856484D 02	0.103480D 02	0.662695D-01
71	0.171251D 02	0.301848D 03	0.307300D-12	0.178360D 02	0.767578D-01
72	0.140702D 02	0.292879D 03	0.105376D 02	0.167110D 02	0.772070D-01
73	0.112853D 02	0.299853D 03	0.288777D 02	0.155860D 02	0.771094D-01
74	0.950536D 01	0.310703D 03	0.432739D 02	0.148590D 02	0.764062D-01
75	0.774640D 01	0.328817D 03	0.559861D 02	0.141320D 02	0.753125D-01
76	0.606420D 01	0.326266D 03	0.632255D 02	0.134150D 02	0.752734D-01
77	0.448194D 01	0.328199D 03	0.724202D 02	0.126980D 02	0.750781D-01
78	0.220067D 01	0.357466D 03	0.758258D 02	0.116250D 02	0.728516D-01
79	0.0	0.401487D 03	0.797238D 02	0.105530D 02	0.690625D-01
80	0.171251D 02	0.338282D 03	0.0	0.178360D 02	0.775781D-01
81	0.110657D 02	0.357678D 03	0.267016D 02	0.156420D 02	0.771094D-01
82	0,742061D 01	0.358663D 03	0.434724D 02	0.142280D 02	0.769141D-01
83	0.420615D 01	0.347499D 03	0.534363D 02	0.128520D 02	0.770312D-01
84	0.0	0.390797D 03	0.776011D 02	0.108310D 02	0.755273D-01
85	0.171251D 02	0.312169D 03	0.567579D 00	0.178360D 02	0.786328D-01
86	0.139682D 02	0.316128D 03	-0.126945D 03	0.167670D 02	0.780078D-01
87	0.109162D 02	0.335360D 03	-0.277874D 03	0.156970D 02	0.755859D-01
88	0.902395D 01	0.324992D 03	-0.331898D 03	0.150110D 02	0.747656D-01
89	0.727694D 01	0.322973D 03	-0.392978D 03	0.143250D 02	0.733594D-01
90	0.567134D 01	0.320579D 03	-0.356204D 03	0.136650D 02	0.743164D-01
91	0.414305D 01	0.319875D 03	-0.318186D 03	0.130050D 02	0.751562D-01
92	0.203268D 01	0.333436D 03	-0.163312D 03	0.120570D 02	0.773047D-01
93	0.0	0.357598D 03	0.321838D 01	0.111090D 02	0.776172D-01

94	0.171251D 02	0.101387D 03	0.137034D-06	0.178390D 02	0.815625D-01
95	0.154134D 02	0.250834D 03	-0.272363D 01	0.159490D 02	0.798047D-01
96	0.122799D 02	0.434998D 03	0.547570D 01	0.144130D 02	0.756250D-01
97	0.750549D 01	0.718042D 03	-0.510535D 01	0.129000D 02	0.680078D-01
98	0.0	0.806328D 03	0.702657D 01	0.111400D 02	0.666016D-01
99	0.171251D 02	0.325522D 03	-0.591858D 00	0.178360D 02	0.783594D-01
100	0.145999D 02	0.328140D 03	0.128272D 03	0.170180D 02	0.777344D-01
101	0.121933D 02	0.337292D 03	0.274692D 03	0.162000D 02	0.756250D-01
102	0.981814D 01	0.342601D 03	0.328501D 03	0.153500D 02	0.744531D-01
103	0.757093D 01	0.350509D 03	0.393343D 03	0.145000D 02	0.727734D-01
104	0.545171D 01	0.346510D 03	0.362088D 03	0.136500D 02	0.736133D-01
105	0.346047D 01	0.342781D 03	0.329248D 03	0.128000D 02	0.744141D-01
106	0.167140D 01	0.329964D 03	0.170334D 03	0.119850D 02	0.773047D-01
107	0.0	0.325522D 03	0.274369D 01	0.111700D 02	0.783594D-01
NODE	WRL(I)	HT	VT	WT	
1	0.0	0.119851D 03	0.0	0.0	
2	0.0	0.119851D 03	0.0	0.0	
3	0.0	0.119851D 03	0.0	0.0	
4	0.0	0.119851D 03	0.0	0.0	
5	0.0	0.119851D 03	0.0	0.0	
6	0.0	0.119851D 03	0.0	0.0	
7	0.0	0.119851D 03	0.0	0.0	
8	0.0	0.119851D 03	0.0	0.0	
9	0.0	0.119851D 03	0.0	0.0	
10	0.0	0.120023D 03	0.0	0.0	
11	0.0	0.119931D 03	0.0	0.0	
12	0.0	0.119892D 03	0.0	0.0	
13	0.0	0.119866D 03	0.0	0.0	
14	0.0	0.119882D 03	0.0	0.0	
15	0.0	0.119997D 03	0.0	0.0	
16	0.0	0.119983D 03	0.0	0.0	
17	0.0	0.119979D 03	0.0	0.0	
18	0.0	0.119955D 03	0.0	0.0	
19	0.0	0.119935D 03	0.0	0.0	
20	0.0	0.119925D 03	0.0	0.0	
21	0.0	0.119876D 03	0.0	0.0	
22	0.0	0.119885D 03	0.0	0.0	
23	0.0	0.119901D 03	0.0	0.0	
24	0.0	0.120006D 03	0.0	0.0	
25	0.0	0.119972D 03	0.0	0.0	
26	0.0	0.119963D 03	0.0	0.0	
27	0.0	0.119961D 03	0.0	0.0	
28	0.0	0.119825D 03	0.0	0.0	
29	0.0	0.120010D 03	0.0	0.0	
30	0.0	0.119956D 03	0.0	0.0	
31	0.0	0.119990D 03	0.0	0.0	
32	0.0	0.120003D 03	0.0	0.0	
33	0.0	0.120028D 03	0.0	0.0	

34	0.0	0.119993D 03	0.0	0.0
35	0.0	0.119988D 03	0.0	0.0
36	0.0	0.119861D 03	0.0	0.0
37	0.0	0.119707D 03	0.0	0.0
38	0.0	0.119575D 03	0.0	0.0
39	0.0	0.119900D 03	0.0	0.0
40	0.0	0.120313D 03	0.0	0.0
41	0.0	0.120533D 03	0.0	0.0
42	0.0	0.120465D 03	0.0	0.0
43	0.129699D 06	0.109679D 03	0.0	0.102067D 03
44	0.947787D 05	0.111526D 03	0.0	0.192627D 03
45	0.576551D 05	0.114350D 03	0.0	0.312498D 03
46	0.368009D 05	0.116541D 03	0.0	0.358696D 03
47	0.164118D 05	0.119299D 03	0.0	0.418559D 03
48	0.479684D 04	0.121196D 03	0.0	0.450019D 03
49	-0.459027D 04	0.123064D 03	0.0	0.477102D 03
50	-0.933004D 04	0.124129D 03	0.0	0.470508D 03
51	-0.147868D 05	0.125822D 03	0.0	0.482189D 03
52	0.130508D 06	0.109680D 03	0.0	0.813004D 02
53	0.553500D 05	0.113812D 03	0.0	0.322493D 03
54	0.131484D 05	0.119056D 03	0.0	0.441486D 03
55	-0.605129D 03	0.123089D 03	0.0	0.452362D 03
56	0.179205D 04	0.125821D 03	0.0	0.351013D 03
57	0.122148D 06	0.109680D 03	0.0	0.110297D 03
58	0.891297D 05	0.111687D 03	0.0	0.205813D 03
59	0.568869D 05	0.114526D 03	0.0	0.312528D 03
60	0.417927D 05	0.116814D 03	0.0	0.332468D 03
61	0.264309D 05	0.119572D 03	0.0	0.364058D 03
62	0.244574D 05	0.121630D 03	0.0	0.328944D 03
63	0.221581D 05	0.123272D 03	0.0	0.296275D 03
64	0.295863D 05	0.124241D 03	0.0	0.194869D 03
65	0.356163D 05	0.125821D 03	0.0	0.931935D 02
66	0.122132D 06	0.119308D 03	0.570562D 03	0.0
67	0.501321D 05	0.121092D 03	0.264042D 03	0.0
68	0.262210D 05	0.123833D 03	0.156078D 03	0.0
69	0.224053D 05	0.127616D 03	0.151970D 03	0.0
70	0.356137D 05	0.134795D 03	0.286800D 03	0.0
71	0.363756D 05	0.119987D 03	0.169954D 03	0.0
72	0.368875D 05	0.120644D 03	0.183948D 03	0.0
73	0.407767D 05	0.121844D 03	0.218020D 03	0.0
74	0.462860D 05	0.122698D 03	0.259584D 03	0.0
75	0.542189D 05	0.124348D 03	0.319717D 03	0.0
76	0.549649D 05	0.125714D 03	0.341440D 03	0.0
77	0.568844D 05	0.127046D 03	0.373316D 03	0.0
78	0.661521D 05	0.130160D 03	0.474209D 03	0.0
79	0.747823D 05	0.134134D 03	0.590529D 03	0.0
80	0.197087D 05	0.119987D 03	0.920829D 02	0.0
81	0.204085D 05	0.121986D 03	0.108727D 03	0.0

82	0.244303D 05	0.124673D 03	0.143088D 03	0.0
83	0.265507D 05	0.127365D 03	0.172157D 03	0.0
84	0.326492D 05	0.134132D 03	0.251202D 03	0.0
85	0.0	0.119987D 03	0.0	0.0
86	0.0	0.120783D 03	0.0	0.0
87	0.0	0.122067D 03	0.0	0.0
88	0.0	0.123223D 03	0.0	0.0
89	0.0	0.124821D 03	0.0	0.0
90	0.0	0.126075D 03	0.0	0.0
91	0.0	0.127436D 03	0.0	0.0
92	0.0	0.130339D 03	0.0	0.0
93	0.0	0.134132D 03	0.0	0.0
94	0.0	0.120279D 03	0.0	0.0
95	0.0	0.122120D 03	0.0	0.0
96	0.0	0.125018D 03	0.0	0.0
97	0.0	0.127731D 03	0.0	0.0
98	0.0	0.132648D 03	0.0	0.0
99	0.0	0.120854D 03	0.0	0.0
100	0.0	0.121376D 03	0.0	0.0
101	0.0	0.122195D 03	0.0	0.0
102	0.0	0.123484D 03	0.0	0.0
103	0.0	0.125192D 03	0.0	0.0
104	0.0	0.126543D 03	0.0	0.0
105	0.0	0.128131D 03	0.0	0.0
106	0.0	0.129812D 03	0.0	0.0
107	0.0	0.131572D 03	0.0	0.0

APPENDIX D

CALCULATION OF ROTOR ELEMENT FLOW ANGLES

The following is a brief synopsis of the procedure contained in Ref.13 for calculating the outlet relative flow angles in a rotor element from the given inlet relative flow angle and blade solidity. The reader is referred to Ref.13, Chapter VI, for specific details of low speed correlation data.

As stated in Section III.A, uniform flow conditions at the rotor blade edges were assumed. This assumption coupled with knowledge of the mass flow rate and rotational speed, enables one to calculate the inlet relative flow angle, β_1, as shown in Fig 18.

From blade geometry information, the blade solidity, σ,

$$\sigma = \frac{c}{s} \tag{1}$$

is obtained. At this point, β_1, and σ are given and one may calculate β_2, the rotor outlet relative flow angle from correlation curves depicted in Ref 13. The equation used to determine β_2, is the following,

$$\beta_2 = K_2 + \delta \tag{2}$$

where K_2 is the angle between the tangent to the blade mean

camber line and the axial direction (Fig 18). This is obtained from the blade geometry data. δ is the low speed deviation angle which is obtained from the correlation curves in Ref. 13. The following equations show the relationship between δ and the correlation data.

$$\delta = \delta_o^\circ + m\phi \tag{3}$$

$$\delta_o^\circ = (K\delta)_{sh}(K\delta)_t(\delta_o)_{10} \tag{4}$$

The variables $m, K\delta)_{sh}, K\delta)_t$ and $\delta_o)_{10}$, are all values which are obtained from the correlation curves and are all functions of the given blade geometry. The quantity, ϕ, is the blade camber angle and again is obtained from the blade geometry data. Once all the variables are obtained from the correlation data, equation (4) is solved for the deviation angle for an uncambered blade section, δ_o, and then equation (3) is solved for the deviation angle, δ. One now calculates β_2 from equation (2) for the blade element. With β_2 now a known quantity, one now calculates the absolute flow angle, α_2, from uniform flow assumptions.

An example follows for node numbers 43 and 57 (Fig 6). From Ref. [12], Table II, the following quantities are obtained assuming the angle of incidence, i, (Fig 18) is zero and therefore the inlet relative flow angle, β_1, is equal to k_1.

$$\beta_1 = k_1 = 61.88^\circ$$

$$\sigma = 1.3062$$

$$\phi = 6.95^\circ$$

$K_2 = 54.93^\circ$

$\frac{t}{c})$ max = 0.035

tip radius = 18.25 in

hub radius = 9.125 in

Assuming uniform flow at the rotor inlet and a rotational speed of 4359.5 RPM, the following quantities are determined from the rotor inlet velocity diagram (Fig 18).

$$V_m = \frac{\dot{m}}{\rho A} = \frac{(107.6 \text{ lbm/sec})(144 \text{ IN}^2/\text{Ft}^2)}{(0.08 \text{ lbm/Ft}^3)\,\pi\,(18.25^2 - 9.125^2)\text{ IN}^2}$$

V_m = 246.802 Ft/sec

where the area A, is determined from the hub and tip radii and the density is assumed to be 0.08 lbm/cu ft.

Now one is ready to obtain the correlation data. From Ref.[13], Fig 162, with $\beta_1 = 61.88^\circ$ and $\sigma = 1.3062$,

$\delta_o)_{10} = 2.50^\circ$

From Ref.[13], Fig 162, with $\beta_1 = 61.88^\circ$ and $\sigma = 1.3062$,

m = 0.235

From Ref.[13], Fig 172, with t/c) max = 0.0350,

$k\delta)_t = 0.29$

From Ref.[13], page 222, one uses the following value of $(K\delta)_{sh}$ for 65-series blades,

$K\delta)_{sh} = 1.0$

At this point all the necessary data has been obtained for equations (3) and (4),

$$\delta_o^\circ = (1.0)(0.29)(2.50) = 0.725^\circ$$

From equation (3),

$$\delta = 0.725^\circ + 0.235(6.95^\circ) = 2.36^\circ$$

Finally, equation (2) gives the desired value of β_2,

$$\beta_2 = 54.93^\circ + 2.36^\circ = 57.29^\circ$$

At this point the relative flow angle for node 57 has been obtained, $\beta_2 = 57.29^\circ$. These two values of relative flow angles, $\beta_1 = 61.88^\circ$ for node 43 and $\beta_2 = 57.29^\circ$ for node 57, are then read in the program as input data for numerical computation.

This process is repeated at each required blade element section for the proper outlet relative flow angle.

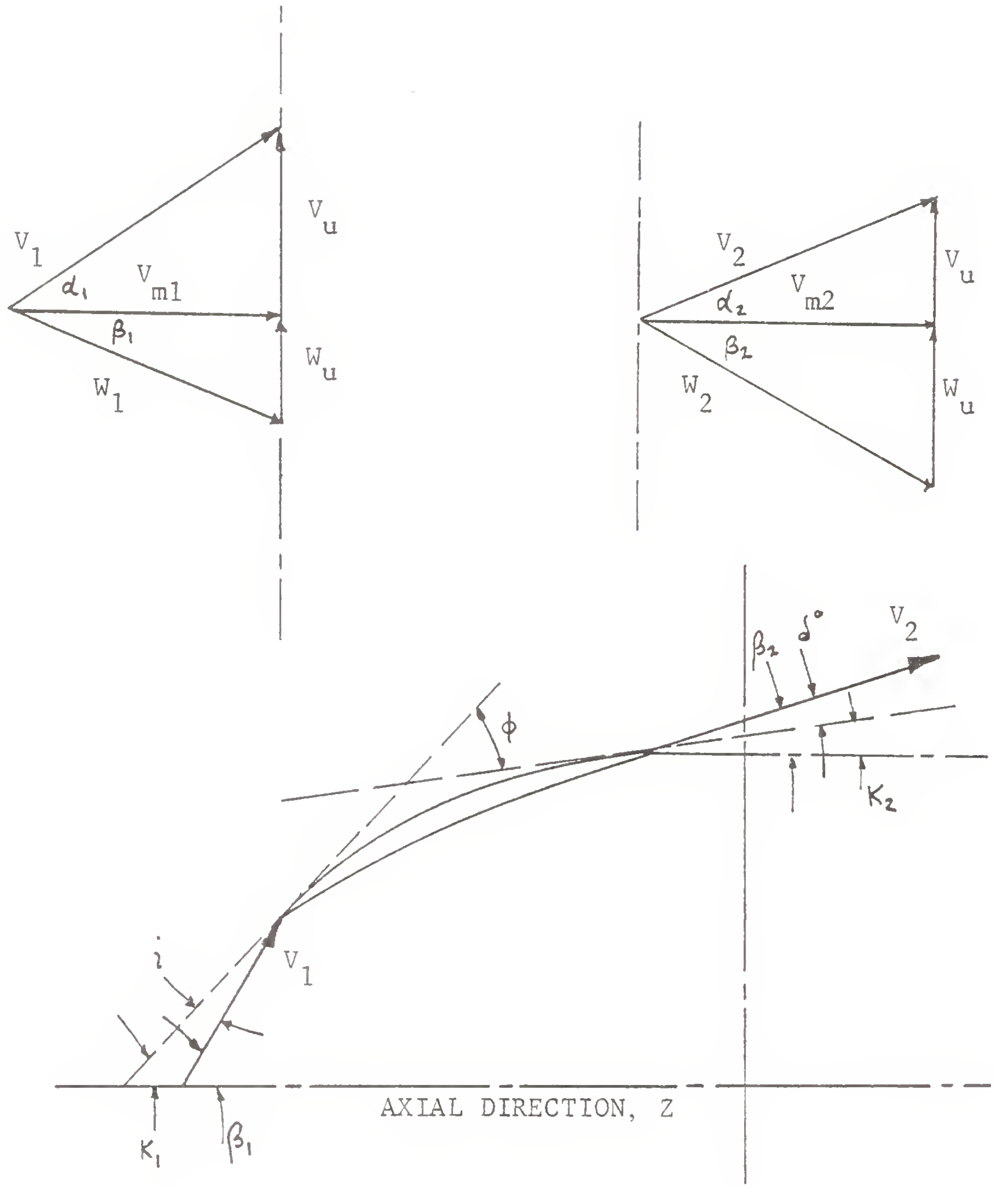

Figure 18 - NOMENCLATURE FOR CASCADE BLADE

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